A Spatial Theory of Overlapping Local Governments*

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Abstract

Local governments in the United States are vertically differentiated. A typical location is served by multiple overlapping jurisdictions that share property tax base and specialize in the provision of one or more local public goods. This paper evaluates the implications of such vertical differentiation for the equilibrium levels of government spending, property tax rates, and household welfare. I propose a spatial theory of overlapping jurisdictions in which residents collectively determine the local mix of expenditures and taxes. Because fiscal policy capitalizes into housing prices and all jurisdictions draw revenue from housing, the cost of raising expenditures in a location is implicitly shared with other coexisting jurisdictions. In equilibrium, this induces higher levels of government spending, higher property tax rates, and lower household welfare compared to scenarios in which jurisdictions are vertically coterminous or only horizontally differentiated.

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1 Introduction

Local governments provide essential public services in the United States. Their scope ranges from K-12 education to fire protection, emergency medical services, utilities, parks and recreation, water conservation, and police protection. Because local governments primarily fund these services through residential property taxes[1], economists and policymakers have long been interested in the implications of local financing for household sorting (Tiebout 1956), housing values (Oates 1969, Hamilton 1976), and inequality in access to public goods (Bucovetsky 1982).

Standard spatial equilibrium models of local jurisdictions consider single-layer, general purpose governments that finance a bundle of public goods with a uniform tax rate on housing expenditures (Ellickson 1971, Hamilton 1975, Stiglitz 1977, Westhoff 1977, Brueckner 1979a, Brueckner 1979b, Brueckner 1979c, Rose-Ackerman 1979, Brueckner 1983, Epple et al. 1984, Epple and Romer 1991, Epple and Platt 1998, Epple and Sieg 1999, Brueckner 2000, Epple et al. 2001, Calabrese et al. 2006, Epple et al. 2010, Calabrese et al. 2012, Brueckner 2023). In the United States, however, local governments are both horizontally and vertically differentiated. They are horizontally differentiated because a public good, such as K-12 education, is typically provided by multiple competing jurisdictions, such as school districts. Local governments are also vertically differentiated because any location is generally served by multiple jurisdictions, each of which delivers one or more services and sets a property tax rate to finance them. Figure 1 shows that the vertical differentiation of local governments is quantitatively important, especially in the Midwest and Pacific regions.

The goal of this paper is to study how such vertical differentiation affects the provision of local public goods and the taxation of residential property. To do so, I develop a spatial equilibrium model of a metropolitan area in which local jurisdictions overlap and thus share tax base. Within each jurisdiction, residents with heterogeneous preferences for public goods vote on their preferred mix of expenditures and taxes. In the model, any change in local fiscal policy is capitalized into housing values. Because jurisdictions share part of their territory and tax the same asset, a change in government spending in a district affects the tax base of all overlapping districts, thereby indirectly impacting their fiscal policies. This

[1] In 2022, property taxes made up approximately 69 percent of total local government tax receipts (U.S. Bureau of Economic Analysis 2023a).

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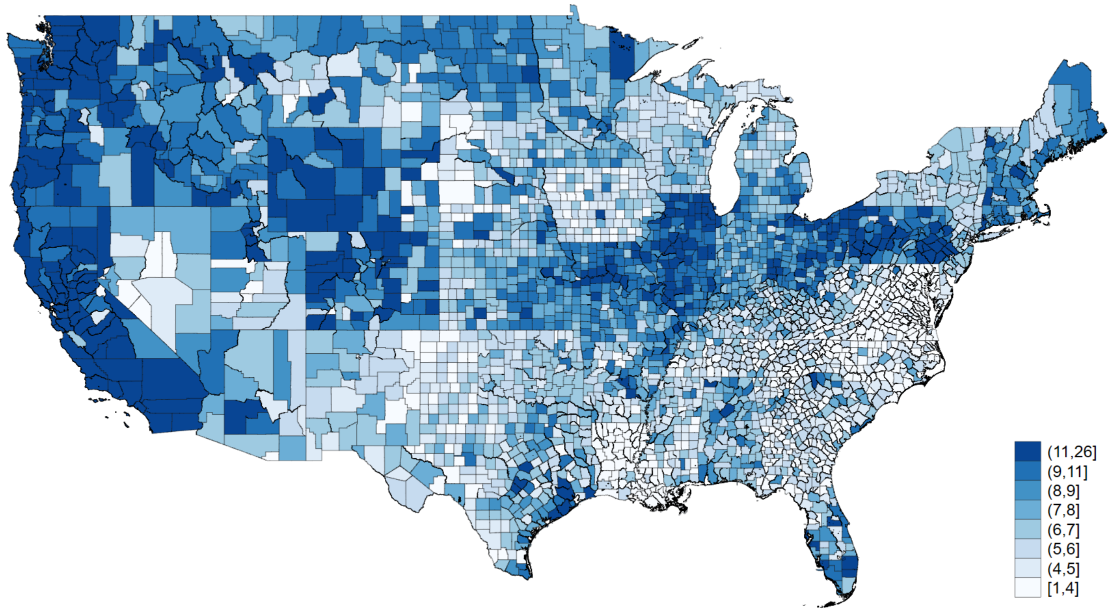
Figure 1: Number of Local Government Types by County



NOTES: This map displays the number of distinct local government types that overlapped in U.S. counties in 2017. Local government "types" are counties, municipalities, townships, school districts, community college districts, fire protection districts, emergency medical services districts, park and recreation districts, as well as several other special purpose districts. Alaska and Hawaii are omitted. Source: author's own calculations based on data from the 2017 Census of Governments (U.S. Census Bureau 2017).

externality is such that a jurisdiction's cost of a marginal increase in spending is borne, in part, by voters who reside outside its boundaries. In equilibrium, this induces a higher level of government spending and higher tax rates relative to a setting in which jurisdictions are perfectly coterminous or do not overlap at all.

The predictions of this model are consistent with the arguments put forward by Berry (2009) and empirically tested by Berry (2008). Both highlight that the vertical structure of local governments induces a fiscal common pool, from which independent overlapping jurisdictions draw more resources than they would if local public goods were provided by single-layer, general purpose governments.

This paper contributes to three literatures. First, it embeds an important feature of the structure of local governments in the United States into a spatial equilibrium model of residential choice in a metropolitan area. As previously discussed, models of equilibria across jurisdictions have a long tradition in public finance, but previous papers abstract from the vertical differentiation of local governments and instead estimate parameters using

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data from towns in Massachusetts, one of the very few states in which local public goods are provided by general purpose, non-overlapping jurisdictions (Epple and Sieg 1999, Epple et al. 2001, Calabrese et al. 2006, Calabrese et al. 2012). Second, this paper adds to the broad literature that studies concurrent taxation by governments sharing tax base. This literature has mostly focused on the interplay between federal and state governments (Johnson 1988, Boadway and Keen 1996, Besley and Rosen 1998, Albouy 2009), whereas local governments have received more limited attention (Greer 2015, Jimenez 2015, Agrawal 2016, Brien and Yan 2020). Finally, this paper leverages tools from modern quantitative spatial modeling (Redding and Rossi-Hansberg 2017 for a review) to analyze equilibria of local jurisdictions.

The remainder of the paper is organized as follows. In Section 2, I provide an overview of local governments in the United States. In Section 3, I illustrate the spatial equilibrium model and its properties. In Section 4, I describe the model solution and perform a number of simulation exercises that offer insights into the welfare implications of alternative local government structures. Section 5 concludes.

2 Local Governments in the United States

In 2022, around 91 thousand local governments spent 1.86 trillion dollars (U.S. Bureau of Economic Analysis 2023a) and employed 12.2 million full-time equivalent units (U.S. Census Bureau 2022). The local government sector, as a whole, employs a workforce that is approximately 40 percent larger than that of federal and state governments combined (U.S. Bureau of Economic Analysis 2023b).

Local governments vary greatly in terms of scope. General purpose jurisdictions, namely counties and municipalities, provide bundles of services, including law enforcement, election organization, urban planning, the court system, and housing assistance. Instead, special purpose jurisdictions, such as fire protection districts, library districts, and water conservation districts, specialize in the provision of a single local public good. As shown in Figure 2, the number of general purpose governments has remained approximately constant over the last eighty years, while special purpose jurisdictions have grown more than fourfold. This growth is attributable to the fact that several State constitutions make it easy for its residents to create local governments (Berry 2008). Local governments vary greatly in terms

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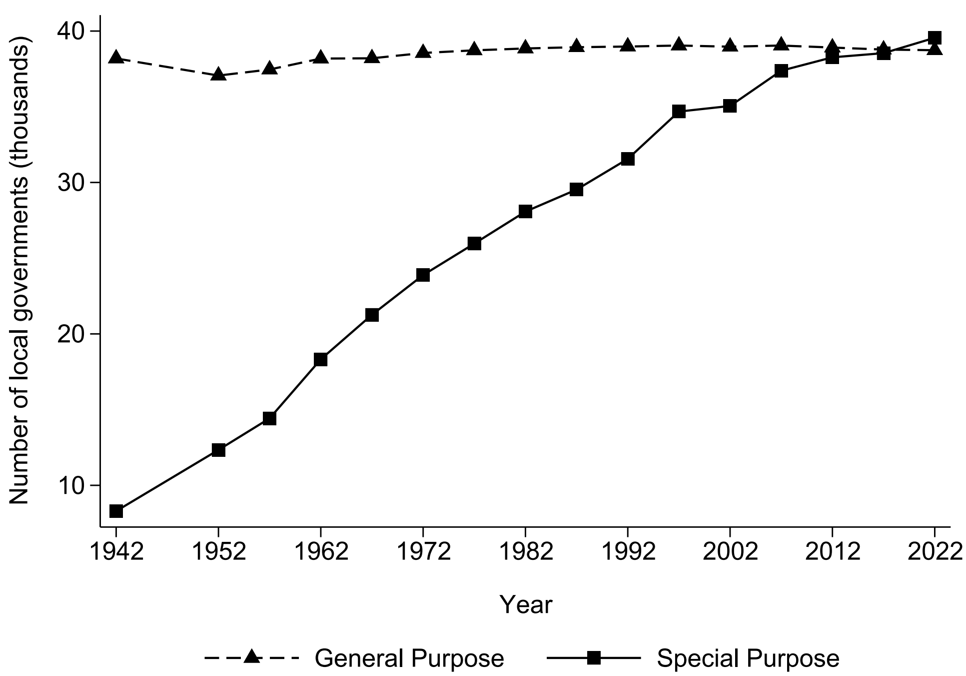
of size too. Special purpose districts can be as large as groups of counties and as small as a few blocks in an urban area. Jurisdiction boundaries are determined at the time of creation, but annexations and secessions are not infrequent.

Local governments primarily fund their services by levying property taxes, sales taxes, and charging residents with fees linked to specific services, such as utilities (U.S. Bureau of Economic Analysis 2023a). Each jurisdiction maintains its own independent budget and determines its intended level of expenditures on an annual basis. County governments are then responsible for regularly assessing property values[2] and computing each jurisdiction's tax rate, i.e., the ratio of its projected expenditures and the aggregate assessed value of residential property within its boundaries. A typical property tax bill lists all of the jurisdictions to which a land parcel is subject to, and the unique combination of local governments overlapping in a given location is referred to as "Tax Code Area" or "Tax Rate Area".

Figure 2: Number of General and Special Purpose Governments in 1942-2022



NOTES: This figure displays the number of general and special purpose governments active in the United States from 1942 to 2022. General purpose jurisdictions include counties, municipalities, and townships. Special purpose districts comprise every other jurisdiction except for school districts. Source: author's own calculations based on data from the 2022 Census of Governments (U.S. Census Bureau 2022).

Finally, local governments are administered by democratically elected representatives or

[2]In most, but not all, states, residential property is appraised annually.

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– in a small number of cases – State-appointed officials. In addition to local elections for selecting representatives, residents frequently participate in referenda, which allow local governments to seek approval for tax increases that administrators alone cannot enact. These referenda have garnered significant attention in the empirical public finance literature that estimates the effect of increased government expenditure on various outcomes, such as student test scores (Cellini et al. 2010, Darolia 2013, Hong and Zimmer 2016, Martorell et al. 2016, Abott et al. 2020, Baron 2022, Rohlin et al. 2022, Baron et al. 2022).

3 A Spatial Equilibrium Model with Overlapping Jurisdictions

In line with prior literature, this model describes a metropolitan area in which households choose where to live, housing prices are determined locally, and the provision of public goods occurs via majority voting. The model is static and is meant to capture long-term allocations of households, government spending, tax rates, and housing prices.

Consider a unit mass of households indexed by i. Households can be partitioned into a finite set of observable types indexed by $k \in \mathcal{K}$, each with mass $\sigma^k \in (0, 1)$. Households choose one among a finite set of localities indexed by $a \in \mathcal{A}$. Public goods are provided by jurisdictions indexed by $j \in \mathcal{J}$ that do not necessarily coincide with localities because jurisdictions of different types overlap arbitrarily. The set of jurisdictions overlapping in community a is denoted with $\mathcal{J}_a$. Symmetrically, the set of areas spanned by jurisdiction j is denoted with $\mathcal{A}_j$. The boundaries of jurisdictions are fixed and the model abstracts from commuting and the labor market. As a matter of fact, income is a type-specific endowment. This choice is consistent with the assumption that firm location choice is not affected by residential property taxation and the structure of local governments. As a consequence, amenities in households' utility function will incorporate the value of location-specific features that can be attributed to the geographic distance between residents and firms.

3.1 Households

The household residential choice problem is similar to Epple and Platt (1998), with one important distinction. In this model, I do not characterize heterogeneity in preferences for

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local public goods by parameterizing the joint probability distribution of household income and taste for public spending. Instead, I leverage a finite set of observable household types that differ in their preference strength for public goods. Moreover, I augment households' utility function with an additive idiosyncratic preference shock for locations. These choices are in line with workhorse models of neighborhood choice in urban economics (Bayer et al. 2007, Ahlfeldt et al. 2015, Almagro and Domínguez-Iino 2024) as well as worker and firm location choice in public finance (Busso et al. 2013, Kline and Moretti 2014, Suárez Serrato and Zidar 2016, Fajgelbaum et al. 2019) and labor economics (Moretti 2011, Moretti 2013, Diamond 2016, Diamond and Gaubert 2017). In area a, households' utility is log-additive in exogenous location amenities A_a, housing floor space H, a composite numeraire consumption good X, and government spending per capita in all of the jurisdictions that overlap in that area $\{G_j\}_{j\in\mathcal{J}_a}$. In addition, the price of the numeraire good is normalized to one and households rent housing space at rate R_a. They also pay property taxes to finance the provision of local public goods. Importantly, the property tax rate in location a is the sum of the rates levied by the jurisdictions that overlap there,

$$\tau_a \equiv \sum_{j\in\mathcal{J}_a} \tau_j \tag{1}$$

Households are endowed with income y^k that is allowed to vary only across types. In any location a, type-k households demand housing space and the numeraire to maximize their utility subject to a budget constraint:

$$\max_{H,X} \left\{ A_a + \sum_{j\in\mathcal{J}_a} \alpha_j^k \log G_j + \beta^k \log H + \gamma^k \log X \right\} \quad \text{s.t.} \quad X + R_a H \left(1+\tau_a\right) \leq y^k \tag{2}$$

Household i's indirect utility stemming from this utility maximization problem is

$$V_{ia} = \rho^k + \sum_{j\in\mathcal{J}_a} \alpha_j^k \log G_j - \beta^k \log R_a - \beta^k \log\left(1+\tau_a\right) + A_{ia} \tag{3}$$

where ρ^k is a deterministic constant. I model the amenity component of utility as the sum of a location-type-specific mean and a random variable that follows a Type-I Extreme Value distribution with type-specific scale parameter θ^k,

$$A_{ia} = \overline{a}_a^k + U_{ia} \quad \text{with} \quad U_{ia} \sim \text{T1EV}\left(0, \theta^k\right) \tag{4}$$

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Households sort into the area that yields the highest indirect utility. As in McFadden (1974), the parametric assumption on the idiosyncratic component of utility implies a closed-form expression for the mass of type-k households who choose location a,

$$N_a^k = \sigma^k \frac{\exp\left(v_a^k/\theta^k\right)}{\sum_{a'} \exp\left(v_{a'}^k/\theta^k\right)} \tag{5}$$

where the nonstochastic component of utility is

$$v_a^k \equiv \rho^k + \overline{a}_a^k + \sum_{j \in \mathcal{J}_a} \alpha_j^k \log G_j - \beta^k \log R_a - \beta^k \log\left(1+\tau_a\right) \tag{6}$$

Jurisdictions primarily differ by their function. As a matter of fact, counties, municipalities, school districts, and special purpose districts deliver mutually exclusive services. Thus, jurisdictions that perform the same function do not overlap. Given a jurisdiction j, such as "Chicago Public Schools", let F be a categorical variable that returns a jurisdiction's function. In this example, $\text{F}(j) = \text{SCHOOL}$. To restrict the cardinality of the set of parameters measuring preferences for local government spending, I assume that the marginal value of any class of public goods (e.g., K-12 education, fire protection, etc.) does not exhibit variation across jurisdictions for a given household type. Formally, for any k,

$$\alpha_j^k = \alpha_{j'}^k \text{ for all } (j, j') \text{ s.t. } \text{F}(j) = \text{F}(j') \tag{7}$$

This restriction implies that α_j^k can be interpreted as the additional utility enjoyed by type-k households due to a marginal change in logged government spending per capita on good j.

3.2 Housing Market

In each area, housing space is supplied competitively. Firms in the construction sector produce with homogeneous technology that exhibits decreasing returns to scale. Thus, the marginal cost of housing space is strictly increasing in the output. For rental rates of housing above the average cost, the housing supply function is

$$\log H_a^{\text{S}} = \lambda + \eta \log R_a + B_a \tag{8}$$

where λ is a deterministic constant, $\eta > 0$ denotes the elasticity of housing supply, and B_a is a random variable that captures idiosyncratic productivity shocks in the construction sector.

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Moreover, the utility maximization and location choice problems jointly yield the aggregate demand for housing in location a,

$$\log H_a^{\mathrm{D}} = \log \sum_{k'} \pi^{k'} N_a^{k'} - \log R_a - \log(1+\tau_a) \tag{9}$$

with $\pi^k \equiv \frac{\beta^k}{\beta^k+\gamma^k} y^k$. The market-clearing rental rate of housing is such that aggregate housing expenditures in equilibrium are

$$\log R_a H_a = \log \sum_{k'} \pi^{k'} N_a^{k'} - \log(1+\tau_a) \tag{10}$$

3.3 Provision of Local Public Goods

Local fiscal policy is determined by jurisdictions, not areas. Jurisdictions choose a level of government spending per capita G and set a property tax rate τ to fund it. Each jurisdiction runs a balanced budget,

$$G_j N_j = \tau_j R_j H_j \iff G_j \sum_{a \in \mathcal{A}_j} N_a = \tau_j \sum_{a \in \mathcal{A}_j} R_a H_a \tag{11}$$

Clearly, for any level of G_j, τ_j is pinned down by population and housing expenditures. The remainder of this section will delve into the collective action process that aggregates preferences to determine a jurisdiction's expenditure-tax mix. First, I will derive household type k's preferred tax rate to fund the provision of public good j in area a. I will then apply a similar argument to compute the tax rate preferred by every other type in all areas spanned by jurisdiction j. Subsequently, I will illustrate that majority-rule voting is sufficient for a unique voting equilibrium to exist in every jurisdiction.

The level of government spending per capita on public good j preferred by type-k households who live in area a is the one that maximizes their indirect utility,

$$G_{ja}^k = \arg\max_{G_j} v_a^k = \arg\max_{G_j} \left\{ \sum_{j \in \mathcal{J}_a} \alpha_j^k \log G_j - \beta^k \log R_a - \beta^k \log(1+\tau_a) \right\} \tag{12}$$

Assuming that the objective function is strictly concave in $\log G_j$ (this is proved in Appendix A.5.3), the first-order condition associated with this maximization problem is

$$\underbrace{\alpha_j^k}_{\text{marginal benefit}} = \underbrace{\beta^k \frac{d \log R_a}{d \log G_j}\bigg|_{G_j = G_{ja}^k} + \beta^k \sum_{j' \in \mathcal{J}_a} \frac{1+\tau_{j'}}{1+\tau_a} \frac{d \log(1+\tau_{j'})}{d \log G_j}\bigg|_{G_j = G_{ja}^k}}_{\text{marginal cost}} \tag{13}$$

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Intuitively, the marginal benefit of an increase in government spending is its marginal utility. On the other hand, the marginal cost of an increase in government spending is the marginal disutility that stems from an increase in the local gross-of-tax rental rate of housing required to finance it. Clearly, R_a and $\{\tau_{j'}\}_{j' \in \mathcal{J}_a}$ are endogenous variables and their values are constrained by two restrictions, namely housing market clearing and balanced budget. Following Epple and Romer (1991), these equations define a Government Possibility Frontier (GPF), a relationship between government spending and the gross-of-tax rental rate of housing along which any spending change is such that the two constraints hold. Because a voter in location a belongs to $|\mathcal{J}_a|$ jurisdictions, each public good is associated with a distinct Government Possibility Frontier. Moreover, each GPF takes into account several constraints jointly. Consider a resident of area a choosing their preferred level of government spending per capita in jurisdiction j. In the remainder of this section, the maintained assumption is that voters internalize the effect of a change in a jurisdiction's expenditure on both area a's housing market and the budget of all jurisdictions that belong to $\mathcal{J}_a$. However, they take as given the housing market in other communities and the fiscal policy chosen by other local governments. As a consequence, the implicit choice variables for a resident of area a voting in jurisdiction j are $\left\{G_j, R_a, \{\tau_{j'}\}_{j' \in \mathcal{J}_a}\right\}$. By assumption, $\left\{\{G_{j'}\}_{j' \neq j}, \{R_a\}_{a' \neq a}, \{\tau_{j'}\}_{j' \notin \mathcal{J}_a}\right\}$ are held constant in the derivations that follow. The $\mathcal{J}_a + 1$ equations characterizing the feasible allocations of $\left\{G_j, R_a, \{\tau_{j'}\}_{j' \in \mathcal{J}_a}\right\}$ are area a's housing market clearing and the balanced budget for each jurisdiction in $\mathcal{J}_a$:

$$J_a\left(G_j, R_a, \{\tau_{j'}\}_{j' \in \mathcal{J}_a}\right) \equiv H_a^{\mathrm{S}} - H_a^{\mathrm{D}} = 0 \tag{14}$$

$$K_j\left(G_j, R_a, \{\tau_{j'}\}_{j' \in \mathcal{J}_a}\right) \equiv \tau_j R_j H_j - G_j N_j = 0 \quad \text{for all } j \in \mathcal{J}_a \tag{15}$$

To derive the slope of the GPF, I proceed analogously to Epple and Romer (1991) and totally differentiate the system of equations around its $\mathcal{J}_a + 2$ arguments:

$$\frac{\partial J_a}{\partial \log G_j} d\log G_j + \frac{\partial J_a}{\partial \log R_a} d\log R_a + \sum_{j' \in \mathcal{J}_a} \frac{\partial J_a}{\partial \log\left(1+\tau_{j'}\right)} d\log\left(1+\tau_{j'}\right) = 0 \tag{16}$$

$$\frac{\partial K_{j'}}{\partial \log G_j} d\log G_j + \frac{\partial K_{j'}}{\partial \log R_a} d\log R_a + \sum_{j' \in \mathcal{J}_a} \frac{\partial K_{j'}}{\partial \log\left(1+\tau_{j'}\right)} d\log\left(1+\tau_{j'}\right) = 0 \tag{17}$$

where equation (17) must hold for every $j' \in \mathcal{J}_a$. To develop intuition on this system, it is useful to specialize the metropolitan area into a partition of four areas implied by two school

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districts and two cities.

3.3.1 The GPF in a 2×2 Metropolitan Area

Consider a stylized metropolitan area with two cities and two school districts that partition the territory into four areas. Using the notation from the model, this metropolitan area comprises four jurisdictions indexed by $j \in \{\textsc{s}_1, \textsc{s}_2, \textsc{c}_1, \textsc{c}_2\}$ and four areas indexed by $a \in \{1, 2, 3, 4\}$.

Figure 3: A Metropolitan Area with two School Districts and two Cities

$a = 1$ $\textsc{s}_1$ and $\textsc{c}_1$	$a = 2$ $\textsc{s}_2$ and $\textsc{c}_1$
$a = 3$ $\textsc{s}_1$ and $\textsc{c}_2$	$a = 4$ $\textsc{s}_2$ and $\textsc{c}_2$

Notes: This figure displays a stylized metropolitan area served by two school districts ($\textsc{s}_1$ and $\textsc{s}_2$) and two cities ($\textsc{c}_1$ and $\textsc{c}_2$) that overlap into four areas indexed by $a \in \{1, 2, 3, 4\}$.

In the remainder of this section, I will derive the Government Possibility Frontier faced by a voter in area $a = 1$, who must determine their preferred level of government spending for jurisdictions $\textsc{s}_1$ and $\textsc{c}_1$. To keep notation compact, I will refer to these jurisdictions as $\textsc{s}$ and $\textsc{c}$, respectively. The school district spans areas 1 and 3, while the city spans areas and 1 and 2. As a consequence, the system of equations that restricts the set of feasible allocations for $G_{\textsc{s}}$ and $G_{\textsc{c}}$ is the following:

$$J_1 \equiv \lambda_1 + (1 + \eta) \log R_1 + B_1 + \log (1 + \tau_{\textsc{s}} + \tau_{\textsc{c}}) - \log \sum_{k'} \pi^{k'} N_1^{k'} = 0 \tag{18}$$

$$K_{\textsc{s}} \equiv \log \tau_{\textsc{s}} + \log (R_1 H_1 + R_3 H_3) - \log G_{\textsc{s}} - \log (N_1 + N_3) = 0 \tag{19}$$

$$K_{\textsc{c}} \equiv \log \tau_{\textsc{c}} + \log (R_1 H_1 + R_2 H_2) - \log G_{\textsc{c}} - \log (N_1 + N_2) = 0 \tag{20}$$

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Consider the goal of deriving the Government Possibility Frontier associated with the preferred choice of G_{S}. Derivations for G_{C} are symmetric. Total differentiation of the system of equations in (18) and (19) around its four arguments yields another system of equations, here presented in matrix form:

$$\begin{bmatrix} J_{1g_{\mathrm{S}}} & J_{1r_1} & J_{1\tau_{\mathrm{S}}} & J_{1\tau_{\mathrm{C}}} \\ K_{\mathrm{S}g_{\mathrm{S}}} & K_{\mathrm{S}r_1} & K_{\mathrm{S}\tau_{\mathrm{S}}} & K_{\mathrm{S}\tau_{\mathrm{C}}} \\ K_{\mathrm{C}g_{\mathrm{S}}} & K_{\mathrm{S}r_1} & K_{\mathrm{S}\tau_{\mathrm{S}}} & K_{\mathrm{C}\tau_{\mathrm{C}}} \end{bmatrix} \begin{bmatrix} dg_{\mathrm{S}} \\ dr_1 \\ d\tau_{\mathrm{S}} \\ d\tau_{\mathrm{C}} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{21}$$

where the unknowns are defined as $dg_{\mathrm{S}} \equiv d\log G_{\mathrm{S}}$, $dr_1 \equiv d\log R_1$, $d\tau_{\mathrm{S}} \equiv d\log(1+\tau_{\mathrm{S}})$, and $d\tau_{\mathrm{C}} \equiv d\log(1+\tau_{\mathrm{C}})$. The matrix of known coefficients is the Jacobian associated with the housing market clearing and balanced budget equations. The system in (21) has three equations and four unknowns. Since the focus of this analysis is on a marginal change in school district spending, one can divide every equation by dg_{S}, thus reducing the number of unknowns by one. The system of equations can then be rewritten as

$$\begin{bmatrix} J_{1r_1} & J_{1\tau_{\mathrm{S}}} & J_{1\tau_{\mathrm{C}}} \\ K_{\mathrm{S}r_1} & K_{\mathrm{S}\tau_{\mathrm{S}}} & K_{\mathrm{S}\tau_{\mathrm{C}}} \\ K_{\mathrm{S}r_1} & K_{\mathrm{S}\tau_{\mathrm{S}}} & K_{\mathrm{C}\tau_{\mathrm{C}}} \end{bmatrix} \begin{bmatrix} dr_1/dg_{\mathrm{S}} \\ d\tau_{\mathrm{S}}/dg_{\mathrm{S}} \\ d\tau_{\mathrm{C}}/dg_{\mathrm{S}} \end{bmatrix} = \begin{bmatrix} -J_{1g_{\mathrm{S}}} \\ -K_{\mathrm{S}g_{\mathrm{S}}} \\ -K_{\mathrm{C}g_{\mathrm{S}}} \end{bmatrix} \tag{22}$$

If the coefficient matrix is nonsingular, the solution to this system yields the desired slopes of the Government Possibility Frontier. For a voter residing in community $a = 1$ choosing their preferred level of school spending, the relevant derivatives are those appearing in the first-order condition (13), namely dr_1/dg_{S}, $d\tau_{\mathrm{S}}/dg_{\mathrm{S}}$, and $d\tau_{\mathrm{C}}/dg_{\mathrm{S}}$. Symmetric derivations for city government spending yield dr_1/dg_{C}, $d\tau_{\mathrm{S}}/dg_{\mathrm{C}}$, and $d\tau_{\mathrm{C}}/dg_{\mathrm{C}}$. The resulting system of first-order conditions for a type-k household in area 1 is

$$\alpha_{\mathrm{S}}^k = \beta^k \left(\frac{dr_1}{dg_{\mathrm{S}}} + \frac{1+\tau_{\mathrm{S}}}{1+\tau_{\mathrm{S}}+\tau_{\mathrm{C}}} \frac{d\tau_{\mathrm{S}}}{dg_{\mathrm{S}}} + \frac{1+\tau_{\mathrm{C}}}{1+\tau_{\mathrm{S}}+\tau_{\mathrm{C}}} \frac{d\tau_{\mathrm{C}}}{dg_{\mathrm{S}}} \right) \tag{23}$$

$$\alpha_{\mathrm{C}}^k = \beta^k \left(\frac{dr_1}{dg_{\mathrm{C}}} + \frac{1+\tau_{\mathrm{S}}}{1+\tau_{\mathrm{S}}+\tau_{\mathrm{C}}} \frac{d\tau_{\mathrm{S}}}{dg_{\mathrm{C}}} + \frac{1+\tau_{\mathrm{C}}}{1+\tau_{\mathrm{S}}+\tau_{\mathrm{C}}} \frac{d\tau_{\mathrm{C}}}{dg_{\mathrm{C}}} \right) \tag{24}$$

where, by definition, $\tau_1 \equiv \tau_{\mathrm{S}} + \tau_{\mathrm{C}}$. Equations (23) and (24) are optimality conditions that jointly characterize a household's preferred levels of government spending on school and city services. This system of two equations in two unknowns, τ_{S} and τ_{C}, can be solved to compute the unique school and city property tax rates preferred by type-k households in area 1. Similar arguments are employed to determine the optimal tax rates for all other groups and locations.

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3.3.2 Majority-Rule Voting

To determine the property tax rate collectively chosen in each jurisdiction, I assume residents vote with majority rule. An appealing feature of this model is that, despite the overlapping structure of local governments, voters implicitly participate to multiple one-dimensional elections. As a matter of fact, each jurisdiction independently sets its fiscal policy. Moreover, every household type has an area-jurisdiction-specific preferred tax rate, τ_{ja}^{k}, and this policy variable can be ordered within any jurisdiction. The global strict concavity of the objective function ensures that tax rates further away from a group's bliss point are less preferred. Formally, preferences are single-peaked. Single-peaked preferences and voting on a unidimensional policy variable are the two assumptions required for the median voter theorem to hold (Black 1948). Thus, the equilibrium tax rate in jurisdiction j is the median rate among those preferred by its residents. Formally, the collectively chosen rate τ_j is such that

$$\sum_{a' \in \mathcal{A}_j} \sum_{k'} \frac{N_{a'}^{k'}}{N_j} \mathbb{I}\left[\tau_{ja'}^{k'} \leq \tau_j\right] \geq 0.5 \quad \text{and} \quad \sum_{a' \in \mathcal{A}_j} \sum_{k'} \frac{N_{a'}^{k'}}{N_j} \mathbb{I}\left[\tau_{ja'}^{k'} \geq \tau_j\right] \geq 0.5 \tag{25}$$

An analogous argument can be applied to every other jurisdiction in area a to obtain τ_a. Since both the set of locations $\mathcal{A}_j$ and the set of household types $\mathcal{K}$ are finite, the median property tax rate τ_j need not be unique. As a matter of fact, both inequalities on line (25) may hold as equalities, implying that two equilibrium rates exist. In this scenario, τ_j is assumed to be the simple average of the two median rates.

3.3.3 The Equilibrium Tax Rate under Myopic Voting

Without further restrictions, it is hard to provide an economic interpretation to the slope of the Government Possibility Frontier. To develop some intuition on the implications of overlapping jurisdictions for the expenditure-tax mix and ultimately welfare, it is convenient to assume that voters are myopic. Myopic voting is a common restriction in models of voting behavior applied to local jurisdictions (Westhoff 1977, Epple et al. 1984, Calabrese et al. 2012). In this context, as clarified by Epple and Romer (1991), this assumption entails that voters take community boundaries as fixed and ignore any effect of spending changes on household mobility. This restriction can be viewed as weakening the rationality requirements that the model attributes to voters, since it reduces the set of model variables voters based

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their choice on. The main practical implication of this assumption is that, for the purpose of deriving the slope of the Government Possibility Frontier, any partial derivative of N_a^k is set to zero. As a result, in the example of a 2×2 metropolitan area, the system in equation (22) simplifies as follows:

$$\begin{bmatrix} 1+\eta & \frac{1+\tau_{\mathrm{S}}}{1+\tau_1} & \frac{1+\tau_{\mathrm{C}}}{1+\tau_1} \\ (1+\eta)\,\Psi_{1\mathrm{S}} & \frac{1+\tau_{\mathrm{S}}}{\tau_{\mathrm{S}}} & 0 \\ (1+\eta)\,\Psi_{1\mathrm{C}} & 0 & \frac{1+\tau_{\mathrm{C}}}{\tau_{\mathrm{C}}} \end{bmatrix} \begin{bmatrix} dr_1/dg_{\mathrm{S}} \\ d\tau_{\mathrm{S}}/dg_{\mathrm{S}} \\ d\tau_{\mathrm{C}}/dg_{\mathrm{S}} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \tag{26}$$

where $\Psi_{aj} \equiv \frac{R_a H_a}{\sum_{a' \in \mathcal{A}_j} R_{a'} H_{a'}}$ denotes location a's housing expenditure share in jurisdiction j. The resulting components of the Government Possibility Frontier are easily interpretable. To begin with, the total derivative of the rental rate of housing with respect to school spending per capita is

$$\frac{dr_1}{dg_{\mathrm{S}}} = -\frac{1}{1+\eta}\left(\frac{\tau_{\mathrm{S}}}{1+\tau_1-\Psi_{1\mathrm{S}}\tau_{\mathrm{S}}-\Psi_{1\mathrm{C}}\tau_{\mathrm{C}}}\right) < 0 \tag{27}$$

Under myopic voting, a marginal increase in school spending has an unambiguous negative effect on the net-of-tax rental rate of housing because the higher tax rate required to finance it depresses housing demand. Importantly, the magnitude of this effect increases monotonically with $\Psi_{1\mathrm{C}}$, the housing expenditure share of area $a = 1$ within the city. This is a core result. Ceteris paribus, the more a jurisdiction shares tax base with one or more other overlapping jurisdictions, the larger is the implicit negative effect of a local expenditure change on the net-of-tax rental rate of housing. In other words, the vertical differentiation of local governments *amplifies* the effects of a local spending change. Why is this the case? Because the school district shares tax base with the city and, for the city budget to remain balanced, a higher city rate must offset the tax base erosion induced by a fall in r_1. As a matter of fact, the total derivative of the city property tax rate with respect to school spending per capita is

$$\frac{d\tau_{\mathrm{C}}}{dg_{\mathrm{S}}} = \frac{\Psi_{1\mathrm{C}}\tau_{\mathrm{S}}}{1+\tau_1-\Psi_{1\mathrm{S}}\tau_{\mathrm{S}}-\Psi_{1\mathrm{C}}\tau_{\mathrm{C}}} > 0 \tag{28}$$

In addition,

$$\frac{d\tau_{\mathrm{S}}}{dg_{\mathrm{S}}} = \frac{\tau_{\mathrm{S}}}{1+\tau_{\mathrm{S}}} + \frac{\Psi_{1\mathrm{S}}\tau_{\mathrm{S}}}{1+\tau_{\mathrm{S}}}\left(\frac{\tau_{\mathrm{S}}}{1+\tau_1-\Psi_{1\mathrm{S}}\tau_{\mathrm{S}}-\Psi_{1\mathrm{C}}\tau_{\mathrm{C}}}\right) > 0 \tag{29}$$

As expected, a marginal increase in school spending induces a higher school property tax rate. Once again, the steepness of this slope increases with $\Psi_{1\mathrm{C}}$, area $a = 1$'s share of housing

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expenditures in the city. To summarize, an increase in school district expenditures results in a higher property tax rate, not only within the school district itself, but also in the city that overlaps with it. Such propagation occurs as a result of the net-of-tax housing price reduction caused by the local school tax hike. In any area within the school district, the magnitude of this spillover increases with the contribution of that area to the tax revenue received by overlapping cities. Ultimately, since a change in one jurisdiction's fiscal policy affects the tax base shared with other jurisdictions, local residents bear only a fraction of the cost of funding that policy change. More formally, it is useful to revisit the marginal cost on the right side of equation (23). The rate at which this marginal cost increases as a function of τ_{S} is increasing in the share of housing expenditures that the school district shares with the city, i.e.,

$$\frac{\partial^2}{\partial \tau_{\text{S}} \partial \Psi_{1\text{C}}} \left(\frac{dr_1}{dg_{\text{S}}} + \frac{1+\tau_{\text{S}}}{1+\tau_{\text{S}}+\tau_{\text{C}}} \frac{d\tau_{\text{S}}}{dg_{\text{S}}} + \frac{1+\tau_{\text{C}}}{1+\tau_{\text{S}}+\tau_{\text{C}}} \frac{d\tau_{\text{C}}}{dg_{\text{S}}} \right) > 0 \tag{30}$$

In economic terms, a larger $\Psi_{1\text{C}}$ implies that a larger fraction of the marginal cost of increasing school spending is borne by school district residents. In other words, the fiscal externality that this policy change imposes on households who live within city borders, but outside the school district, is smaller. As a consequence, for any $\Psi_{1\text{C}}$ in the interior of the unit interval, and ceteris paribus, school district residents prefer a higher tax rate than they would if the two jurisdictions were vertically coterminous, i.e., $\Psi_{1\text{C}} = 1$. A symmetric argument applies to city residents and the first-order condition in equation (24). In equilibrium, this induces all household types to prefer higher property tax rates.

Replacing the slopes of the Government Possibility Frontier in (27), (28), and (29) into the first-order condition in (24) yields the following implicit expression for the school property tax rate preferred by households of type k residing in area $a = 1$:

$$\alpha_{\text{S}}^k = \beta^k \left(\frac{\tau_{\text{S}}}{1+\tau_1-\Psi_{1\text{S}}\tau_{\text{S}}-\Psi_{1\text{C}}\tau_{\text{C}}} \right) \left(\frac{\Psi_{1\text{S}}\tau_{\text{S}}+\Psi_{1\text{C}}\tau_{\text{C}}}{1+\tau_1} - \frac{1}{1+\eta} \right) + \beta^k \frac{\tau_{\text{S}}}{1+\tau_1} \tag{31}$$

This first-order condition characterizes the best response for type-k households who reside in area $a = 1$ and choose their preferred level of school spending. As a matter of fact, for any city property tax rate τ_{C}, equation (31) returns the utility-maximizing school property tax rate τ_{S}. This best response and its symmetric city counterpart jointly determine the unique

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pair of preferred tax rates $\left(\tau_{\mathrm{S}1}^{k}, \tau_{\mathrm{C}1}^{k}\right)$. Specifically, for $j \in \{\mathrm{S}, \mathrm{C}\}$,

$$\tau_{j1}^{k} = \frac{\alpha_{j}^{k}\left(1+\eta\right)}{\beta^{k}\eta - \left(1+\eta\right)\sum_{\ell\in\{\mathrm{S},\mathrm{C}\}}\alpha_{\ell}^{k}\left(1-\Psi_{1\ell}\right)} \tag{32}$$

Appendix A.5.4 shows that each of these tax rates increases with the strength of the preference for government spending α_{j}^{k}, diminishes with the strength of the preference for housing space β^{k}, and declines with the elasticity of housing supply η. While the first two findings are intuitive, the third is explained by the observation that a more elastic housing supply mitigates the responsiveness of the equilibrium rental rate of housing to changes in property tax rates. As a consequence, a jurisdiction requires a lower rate to increase its expenditure while still maintaining a balanced budget.

3.4 Definition of Equilibrium

An equilibrium consists of a finite set of jurisdictions indexed by $j \in \mathcal{J}$ that overlap into a finite set of areas indexed by $a \in \mathcal{A}$; a unit mass of households indexed by i; a partition of households into observable types indexed by $k \in \mathcal{K}$, each with positive mass σ^{k} and endowed with positive income y^{k}; a partition of households across areas such that each area has positive population N_{a}; a set of stochastic location amenities $\{A_{a}\}_{a}$; a set of stochastic productivity shocks in the residential construction sector $\{B_{a}\}_{a}$; a vector of rental rates of housing $\{R_{a}\}_{a}$ and property tax rates $\{\tau_{j}\}_{j}$; an allocation of government spending per capita $\{G_{j}\}_{j}$; an allocation of housing space $\{H_{i}\}_{i}$ and numeraire consumption good $\{X_{i}\}_{i}$ such that

(1) Households in every area choose housing space and the numeraire consumption good to maximize their utility subject to a budget constraint. For any $a \in \mathcal{A}$,

$$\max_{H,X}\left\{A_{a} + \sum_{j\in\mathcal{J}_{a}}\alpha_{j}^{k}\log G_{j} + \beta^{k}\log H + \gamma^{k}\log X\right\} \quad \text{s.t.} \quad X + R_{a}H\left(1+\tau_{a}\right) \le y^{k}$$

where the aggregate property tax rate is

$$\tau_{a} \equiv \sum_{j\in\mathcal{J}_{a}}\tau_{j}$$

(2) Each household resides in the area that yields the highest indirect utility,

$$V_{ia} = \rho^{k} + \sum_{j\in\mathcal{J}_{a}}\alpha_{j}^{k}\log G_{j} - \beta^{k}\log R_{a} - \beta^{k}\log\left(1+\tau_{a}\right) + A_{ia}$$

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where ρ^k is a deterministic constant and the stochastic valuation of amenities is parameterized as

$$A_{ia} = \overline{a}_a^k + U_{ia} \quad \text{with} \quad U_{ia} \sim \text{T1EV}\left(0, \theta^k\right)$$

(3) Firms in the construction sector supply housing with a technology that exhibits decreasing returns to scale, so that the supply of housing space is, for any a,

$$\log H_a^{\text{S}} \equiv \lambda + \eta \log R_a + B_a$$

(4) The housing market clears in every area. For any a,

$$\log H_a = \log H_a^{\text{S}} = \log H_a^{\text{D}} \equiv \log \sum_{k'} \pi^{k'} N_a^{k'} - \log R_a - \log\left(1 + \tau_a\right)$$

(5) Each jurisdiction operates with a balanced budget. For any j,

$$\log G_j + \log \sum_{a \in \mathcal{A}_j} N_a = \log \tau_j + \log \sum_{a \in \mathcal{A}_j} R_a H_a$$

(6) Each jurisdiction's level of government spending per capita is determined according to majority-rule voting among its residents. For any j, the collectively chosen property tax rate τ_j is such that

$$\sum_{a' \in \mathcal{A}_j} \sum_{k'} \frac{N_{a'}^{k'}}{N_j} \mathbb{I}\left[\tau_{ja'}^{k'} \leq \tau_j\right] \geq 0.5 \quad \text{and} \quad \sum_{a' \in \mathcal{A}_j} \sum_{k'} \frac{N_{a'}^{k'}}{N_j} \mathbb{I}\left[\tau_{ja'}^{k'} \geq \tau_j\right] \geq 0.5$$

where τ_{ja}^k denotes the tax rate preferred by type-k households in area a to finance government spending by jurisdiction $j \in \mathcal{J}_a$.

3.5 Welfare

I compute household welfare by exploiting the parametric assumption on the stochastic component of utility. As in Williams (1977) and Small and Rosen (1981), household type k's welfare is

$$W^k \equiv \mathbb{E}\left[\max_{a \in \mathcal{A}} \left\{v_a^k + A_{ia}\right\}\right] = c + \ln \sum_{a \in \mathcal{A}} \exp\left(\frac{v_a^k}{\theta^k}\right) \tag{33}$$

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where v_a^k is the deterministic component of household type k's utility in area a, the expectation is taken with respect to the probability distribution of A_{ia}, and c denotes the Euler-Mascheroni constant. To determine aggregate welfare, I integrate type-specific welfare over its probability mass function,

$$W \equiv \sum_k \sigma^k W^k \tag{34}$$

4 Model Solution and Simulation

Let $\mathcal{P}$ denote the set of model parameters:

$$\mathcal{P} = \left\{ \left\{ \alpha_j^k \right\}_{j,k}, \left\{ \beta^k \right\}_k, \left\{ \gamma^k \right\}_k, \left\{ \theta^k \right\}_k, \left\{ \sigma^k \right\}_k, \left\{ y^k \right\}_k, \left\{ \overline{a}_a^k \right\}_{a,k}, \lambda, \eta, \left\{ b_a \right\}_a \right\} \tag{35}$$

where b_a indicates a realization of B_a. Furthermore, let $\mathcal{Y}$ denote the set of endogenous variables:

$$\mathcal{Y} = \left\{ \left\{ N_a^k \right\}_{a,k}, \left\{ G_j \right\}_j, \left\{ \tau_j \right\}_j \right\} \tag{36}$$

Other endogenous variables, such as $\left\{ R_a \right\}_a$, can be recovered once $\mathcal{Y}$ is known. Notice that the cardinality of $\mathcal{Y}$ is $|\mathcal{A}| \times |\mathcal{K}| + |\mathcal{J}| + |\mathcal{J}|$. For a given set of parameter values $\mathcal{P}$, I solve the system implied by the following non-redundant equations:

(1) $|\mathcal{A}| \times |\mathcal{K}|$ location-type choice probabilities in (5);

(2) $|\mathcal{J}|$ jurisdiction balanced budgets in (11);

(3) $|\mathcal{J}|$ jurisdiction property tax rates chosen with majority voting in (25).

Future drafts of this paper will include a proof of existence and uniqueness of the solution to this system. In the meantime, I experimented with a large number of possible parameter vectors and initial guesses, always achieving convergence to the same solution.

4.1 Simulation Exercises

In this section, I perform a number of simulation exercises using non-calibrated parameter values. The primary goal of these simulations is to explore the implications of imperfectly overlapping governments for the level of public spending, property tax rates, and household

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Table 1: Parameter Values for Model Simulations

Parameter	**Value**
$\alpha^{\text{a}}_{\text{s}}$, $\alpha^{\text{b}}_{\text{s}}$	.12
$\alpha^{\text{a}}_{\text{c}}$, $\alpha^{\text{b}}_{\text{c}}$	.06
β^{a}, β^{b}	.5
γ^{a}, γ^{b}	.32
θ^{a},θ^{b}	1
y^{a},y^{b}	5
σ^{a}	.51
σ^{b}	.49
$\overline{a}^{\text{a}}_1$, $\overline{a}^{\text{a}}_2$, $\overline{a}^{\text{a}}_3$, $\overline{a}^{\text{a}}_4$	.1
$\overline{a}^{\text{b}}_1$, $\overline{a}^{\text{b}}_2$, $\overline{a}^{\text{b}}_3$, $\overline{a}^{\text{b}}_4$	.1
λ	1
η	1
b_1, b_2, b_3, b_4	1

NOTES: This table reports model parameter values for the simulation exercises described in this section.

welfare. In doing so, I abstract from all other sources of heterogeneity. First, I assume that household types are endowed with the same level of income and have identical preferences for public goods. Second, I assume that the valuation of local amenities is homogeneous both across types and areas. Third, I restrict housing supply parameters to be constant in space. For simplicity, I leverage the stylized metropolitan area depicted in Figure 3 and assume that there are only two household types, $\mathcal{K} = \{\text{a}, \text{b}\}$. To break election ties, one household type has a marginally larger mass. However, this choice is irrelevant for the conclusions of the simulation because preferences and income are homogeneous. Table 1 reports the full list of parameter values.

The goal of the first set of simulations is to compute and compare equilibrium government spending, property tax rates, and welfare in the model with imperfectly overlapping jurisdictions with a similar model in which each city-school district pair has the same tax base. This can be achieved in numerous ways, and I focus on three possible scenarios.

First, I consider a setting in which each area is served by a distinct city-school district pair, implying that eight jurisdictions exist in the metropolitan area as a whole. Second, I focus on the case in which cities and school districts are coterminous and their coverage

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Table 2: Comparison of Model Output across Jurisdiction Structures

Variable	Imperfect	Area	City	Metro
G_{S}	1.08	.85	.85	.85
G_{C}	.54	.43	.43	.43
τ_{S}	.75	.48	.48	.48
τ_{C}	.38	.24	.24	.24
τ_a	1.13	.72	.72	.72
R_a	.21	.24	.24	.24
$R_a\,(1+\tau_a)$	.45	.41	.41	.41
$R_a H_a$	.33	.41	.41	.41
$\tau_a R_a H_a$	.38	.30	.30	.30
W	3.27	3.28	3.28	3.28

NOTES: This table reports the value of selected equilibrium variables from alternative versions of the model. The "Imperfect" column shows the output of the baseline model with imperfectly overlapping jurisdictions. The "Area" column reports the output of a model in which each of the four areas is served by a distinct city-school district pair. The "City" column displays equilibrium variables in a model with perfectly overlapping cities and school districts that follow city boundaries. The "Metro" column shows the output of a model in which public good provision is fully centralized.

areas follow city boundaries. Finally, I consider a version of the model in which public good provision is centralized and the metropolitan area is served by one city and one school district that span its entire territory. Table 2 reports the output of these simulations.

As predicted by the theory, both government spending per capita and tax rates are higher in the model with imperfectly overlapping jurisdictions. Moreover, the net-of-tax rental rate of housing is lower, but accounting for property taxation yields a higher full price of housing. Analogously, the value of the tax base is lower than it would be with perfectly overlapping jurisdictions, but tax revenues are higher. Overall, the effect on aggregate household welfare is negative. Noticeably, the output of the model with perfectly overlapping governments is independent of jurisdiction size. This is explained by the fact that the marginal cost of producing government services is constant – in fact, equal to 1 – and thus the technology in the government sector exhibits constant returns to scale.

The role of economies of scale for the purpose of determining the optimal size of jurisdictions has a long tradition in this literature (Oates 1972 for a comprehensive discussion). Moreover, school district consolidations and municipal annexations occur frequently in present times and are relevant for policy. In the second set of simulations, I introduce

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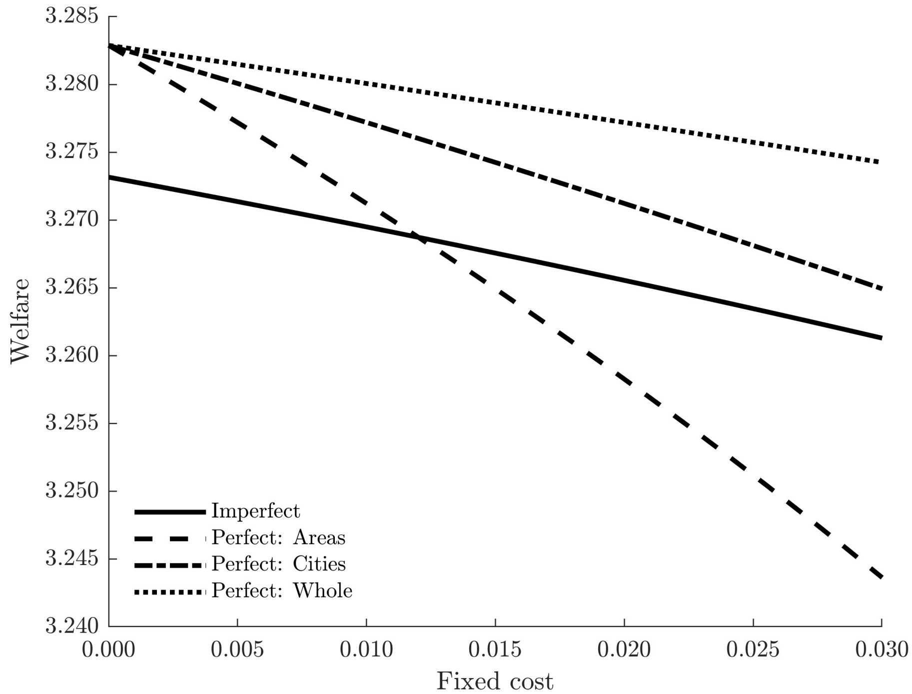
Figure 4: Fixed Costs and Aggregate Welfare



NOTES: This figure displays aggregate welfare against the fixed cost f_j, which is assumed to be homogeneous across jurisdictions. Each of the four lines corresponds to a structure of local governments. "Imperfect" refers to a model with imperfectly overlapping jurisdictions. The other three lines correspond to versions of the model in which jurisdictions overlap perfectly and coincide with areas, cities, or the entire metropolitan area.

increasing returns to scale in the government sector. Specifically, I model the average cost of delivering public goods as

$$c\left(f_j, G_j, N_j\right) = \frac{f_j}{N_j} + G_j \tag{37}$$

where f_j is a deterministic constant that measures fixed costs. If $f_j = 0$, then $c\left(f_j, G_j, N_j\right)$ reduces to G_j and the baseline version of the model is restored. In the presence of fixed costs, a jurisdiction's balanced budget equation becomes

$$c\left(f_j, G_j, N_j\right) N_j = \tau_j R_j H_j \tag{38}$$

I solve the model for each element of a grid of values of f ranging from 0 to 0.03 and for each local government structure described earlier in this section. Figure 4 plots aggregate welfare against the fixed cost. As expected, jurisdiction size matters for aggregate welfare when local governments' production function exhibits increasing returns to scale. Specifically, if $f = 0$, welfare in the model with imperfectly overlapping jurisdictions is lower.

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However, as fixed costs increase, the gains from centralization become larger and eventually the imperfectly overlapping structure produces higher welfare than both the fully decentralized equilibrium and the equilibrium with jurisdictions that coincide with cities. Ultimately, which structure maximizes household welfare is an empirical question.

5 Conclusion

In the United States, local governments are both horizontally and vertically differentiated. As a matter of fact, every location is typically served by multiple overlapping jurisdictions that specialize in the provision of one or more local public goods. This paper has proposed a spatial theory of local governments that overlap and thus share tax base. In the model, each jurisdiction's fiscal policy is collectively determined by voters who differ in their preferences for public goods. Because changes in government spending and property tax rates capitalize into housing values and all jurisdictions draw revenue from housing, a district's fiscal policy affects the tax base of all other overlapping jurisdictions. Voters internalize that they bear only a fraction of the full cost of increasing expenditures in their own jurisdiction, thus facing an incentive to prefer more. In equilibrium, jurisdictions choose a higher level of expenditures and set higher property tax rates than they would if jurisdictions were vertically coterminous or did not overlap at all. In simulation exercises that restrict heterogeneity across households and locations, any alternative local government structure that ensures perfectly overlapping jurisdictions yields higher household welfare.

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Appendix

A Derivations

A.1 Household Utility Maximization

Type-k households face the following utility maximization problem in location a:

$$\max_{H,X} \left\{ A_a + \sum_{j \in \mathcal{J}_a} \alpha^k \log G_j + \beta^k \log H + \gamma^k \log X \right\} \quad \text{s.t.} \quad X + R_a H \left(1 + \tau_a\right) \leq y^k \tag{39}$$

The Lagrangian associated with this maximization problem is

$$\begin{aligned} \mathcal{L}\left(H, X; \lambda_a^k\right) = A_a &+ \sum_{j \in \mathcal{J}_a} \alpha^k \log G_j + \beta^k \log H + \gamma^k \log X \\ &- \lambda_a^k \left(X + R_a H \left(1 + \tau_a\right) - y^k\right) \end{aligned} \tag{40}$$

The first-order necessary conditions are

$$\frac{\partial \mathcal{L}\left(H, X; \lambda_a^k\right)}{\partial H} = \frac{\beta^k}{H_a^k} - \lambda_a^k R_a \left(1 + \tau_a\right) = 0 \tag{41}$$

$$\frac{\partial \mathcal{L}\left(H, X; \lambda_a^k\right)}{\partial X} = \frac{\gamma^k}{X_a^k} - \lambda_a^k = 0 \tag{42}$$

$$\frac{\partial \mathcal{L}\left(H, X; \lambda_a^k\right)}{\partial \lambda_a^k} = -X_a^k - R_a H_a^k \left(1 + \tau_a\right) + y^k = 0 \tag{43}$$

Combining the first two first-order conditions yields

$$\frac{\gamma^k}{X_a^k} = \frac{\beta^k}{R_a \left(1 + \tau_a\right) H_a^k} \iff \frac{\beta^k}{\gamma^k} X_a^k = R_a \left(1 + \tau_a\right) H_a^k \tag{44}$$

Plugging back into the budget constraint,

$$X_a^k + \frac{\beta^k}{\gamma^k} X_a^k = y^k \iff \frac{\beta^k + \gamma^k}{\gamma^k} X_a^k = y^k \iff X_a^k = \frac{\gamma^k}{\beta^k + \gamma^k} y^k \tag{45}$$

Thus,

$$\frac{\beta^k}{\beta^k + \gamma^k} y^k = R_a \left(1 + \tau_a\right) H_a^k \iff H_a^k = \frac{\beta^k}{\beta^k + \gamma^k} \frac{1}{R_a \left(1 + \tau_a\right)} y^k \tag{46}$$

Taking the logarithm of the optimal demand for the numeraire good and housing space,

$$\log X_a^k = \log \left(\frac{\gamma^k}{\beta^k + \gamma^k} \right) + \log y^k \tag{47}$$

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$$\log H_a^k = \log\left(\frac{\beta^k}{\beta^k+\gamma^k}\right) - \log R_a - \log(1+\tau_a) + \log y^k \tag{48}$$

Plugging the Marshallian demands back into the utility function yields household i's indirect utility function:

$$\begin{aligned} V_{ia} = {} & A_{ia} + \sum_{j\in\mathcal{J}_a} \alpha^k \log G_j \\ & + \beta^k \left(\log\left(\frac{\beta^k}{\beta^k+\gamma^k}\right) - \log R_a - \log(1+\tau_a) + \log y^k\right) \\ & + \gamma^k \left(\log\left(\frac{\gamma^k}{\beta^k+\gamma^k}\right) + \log y^k\right) \end{aligned} \tag{49}$$

Define a type-specific deterministic constant:

$$\rho^k \equiv \beta^k \log\left(\frac{\beta^k}{\beta^k+\gamma^k}\right) + \gamma^k \log\left(\frac{\gamma^k}{\beta^k+\gamma^k}\right) + \left(\beta^k+\gamma^k\right)\log y^k \tag{50}$$

Furthermore, recall that household i's valuation of exogenous amenities is $A_{ia} \equiv \overline{a}_a^k + U_{ia}$, with $U_{ia} \sim \text{T1EV}\left(0, \theta^k\right)$. The indirect utility function can thus be re-expressed as follows:

$$V_{ia} = \underbrace{\rho^k + \overline{a}_a^k + \sum_{j\in\mathcal{J}_a} \alpha^k \log G_j - \beta^k \log R_a - \beta^k \log(1+\tau_a)}_{\equiv v_a^k} + U_{ia} \tag{51}$$

where v_a^k indicates the type-location-specific component of utility. Each household chooses the location that maximizes their indirect utility. Because of the parametric assumption regarding the random component of amenity shocks, the probability of choosing location a among type-k households is

$$S_a^k = \frac{\exp\left(v_a^k/\theta^k\right)}{\sum_{a'} \exp\left(v_{a'}^k/\theta^k\right)} \tag{52}$$

Recalling that the mass of type-k households is σ^k, the mass of households who are of type k and sort into location a is

$$N_a^k = \sigma^k S_a^k = \sigma^k \frac{\exp\left(v_a^k/\theta^k\right)}{\sum_{a'} \exp\left(v_{a'}^k/\theta^k\right)} \tag{53}$$

A.2 Equilibrium in the Housing Market

The housing supply equation is

$$\log H_a^S = \lambda + \eta \log R_a + B_a \tag{54}$$

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The aggregate demand for housing among type-k households in location a is

$$H_a^{\mathrm{D},k} = N_a^k H_a^k \tag{55}$$

The aggregate demand for housing in location a can thus be computed as

$$\begin{aligned}
H_a^{\mathrm{D}} &= \sum_{k'} H_a^{\mathrm{D},k'} && (56)\\
&= \sum_{k'} N_a^{k'} H_a^{k'} && (57)\\
&= \sum_{k'} N_a^{k'} \frac{\beta^{k'}}{\beta^{k'}+\gamma^{k'}} \frac{1}{R_a\left(1+\tau_a\right)} y^{k'} && (58)\\
&= \frac{1}{R_a\left(1+\tau_a\right)} \sum_{k'} N_a^{k'} \underbrace{\frac{\beta^{k'}}{\beta^{k'}+\gamma^{k'}} y^{k'}}_{\equiv \pi^{k'}} && (59)\\
&= \frac{\sum_{k'} \pi^{k'} N_a^{k'}}{R_a\left(1+\tau_a\right)} && (60)
\end{aligned}$$

Taking logarithms yields

$$\log H_a^{\mathrm{D}} = \log \sum_{k'} \pi^{k'} N_a^{k'} - \log R_a - \log\left(1+\tau_a\right) \tag{61}$$

The equilibrium rental rate of housing equates log-demand and log-supply of housing:

$$\begin{aligned}
\log H_a^{\mathrm{D}} = \log H_a^{\mathrm{S}} &\iff \lambda + \eta \log R_a + B_a = \log \sum_{k'} \pi^{k'} N_a^{k'} - \log R_a - \log\left(1+\tau_a\right) && (62)\\
&\iff (1+\eta) \log R_a = \log \sum_{k'} \pi^{k'} N_a^{k'} - \log\left(1+\tau_a\right) - \lambda - B_a && (63)\\
&\iff \log R_a = \frac{1}{1+\eta} \log \sum_{k'} \pi^{k'} N_a^{k'} - \frac{1}{1+\eta} \log\left(1+\tau_a\right) - \widetilde{\lambda} - \widetilde{B_a} && (64)
\end{aligned}$$

with $\widetilde{\lambda} \equiv \frac{\lambda}{1+\eta}$ and $\widetilde{B}_a \equiv \frac{B_a}{1+\eta}$. Plugging the equilibrium rental rate of housing into the equation for the log-supply of housing yields the equilibrium level of housing space:

$$\begin{aligned}
\log H_a &= \lambda + \eta \log R_a + B_a && (65)\\
&= \frac{\eta}{1+\eta} \log \sum_{k'} \pi^{k'} N_a^{k'} - \frac{\eta}{1+\eta} \log\left(1+\tau_a\right) - \eta\widetilde{\lambda} - \eta\widetilde{B_a} + \lambda + B_a && (66)\\
&= \frac{\eta}{1+\eta} \log \sum_{k'} \pi^{k'} N_a^{k'} - \frac{\eta}{1+\eta} \log\left(1+\tau_a\right) + \widetilde{\lambda} + \widetilde{B}_a && (67)
\end{aligned}$$

Finally, the equilibrium level of housing expenditure in location j is

$$\log R_a H_a = \log \sum_{k'} \pi^{k'} N_a^{k'} - \log\left(1+\tau_a\right) \tag{68}$$

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A.3 Household Supply

As shown in equation (5), the mass of type-k households who choose to reside in area a is

$$N_a^k = \sigma^k \frac{\exp\left(v_a^k/\theta^k\right)}{\sum_{a'} \exp\left(v_{a'}^k/\theta^k\right)} \tag{69}$$

where the nonstochastic component of utility is

$$v_a^k \equiv \rho^k + \overline{a}_a^k + \sum_{j\in\mathcal{J}_a} \alpha_j^k \log G_j - \beta^k \log R_a - \beta^k \log\left(1+\tau_a\right) \tag{70}$$

Taking logarithms yields

$$\log N_a^k = \log \sigma^k \frac{\exp\left(v_a^k/\theta^k\right)}{\sum_{a'} \exp\left(v_{a'}^k/\theta^k\right)} \tag{71}$$

$$= \log \sigma^k \underbrace{- \log \sum_{a'} \exp\left(v_{a'}^k/\theta^k\right)}_{\equiv \phi^k} + \frac{v_a^k}{\theta^k} \tag{72}$$

$$= \phi^k + \log \sigma^k + v_a^k \tag{73}$$

$$= \phi^k + \underbrace{\log \sigma^k + \rho^k/\theta^k}_{\equiv \zeta^k} + \frac{\overline{a}_a^k}{\theta^k} + \sum_{j\in\mathcal{J}_a} \frac{\alpha_j^k}{\theta^k} \log G_j - \frac{\beta^k}{\theta^k} \log R_a - \frac{\beta^k}{\theta^k} \log\left(1+\tau_a\right) \tag{74}$$

$$= \phi^k + \zeta^k + \frac{\overline{a}_a^k}{\theta^k} + \sum_{j\in\mathcal{J}_a} \frac{\alpha_j^k}{\theta^k} \log G_j - \frac{\beta^k}{\theta^k} \log R_a - \frac{\beta^k}{\theta^k} \log\left(1+\tau_a\right) \tag{75}$$

Computing the exponential again yields

$$N_a^k = \frac{\exp\left(\phi^k + \zeta^k + \frac{\overline{a}_a^k}{\theta^k} + \sum_{j\in\mathcal{J}_a} \frac{\alpha_j^k}{\theta^k} \log G_j\right)}{\exp\left(\frac{\beta^k}{\theta^k} \log R_a + \frac{\beta^k}{\theta^k} \log\left(1+\tau_a\right)\right)} = e^{\phi^k} e^{\zeta^k} \frac{e^{\overline{a}_a^k/\theta^k} \Pi_{j\in\mathcal{J}_a} G_j^{\alpha_j^k/\theta^k}}{R_a^{\beta^k/\theta^k} \left(1+\tau_a\right)^{\beta^k/\theta^k}} \tag{76}$$

Further define

$$\widetilde{\phi}^k \equiv e^{\phi^k} = \exp\left(-\log \sum_{a'} \exp\left(v_{a'}^k/\theta^k\right)\right) = \frac{1}{\sum_{a'} \exp\left(v_{a'}^k/\theta^k\right)} \tag{77}$$

Then the mass of type-k households choosing location a can be expressed as

$$N_a^k = \widetilde{\phi}^k e^{\zeta^k} \frac{e^{\overline{a}_a^k/\theta^k} \Pi_{j\in\mathcal{J}_a} G_j^{\alpha_j^k/\theta^k}}{R_a^{\beta^k/\theta^k} \left(1+\tau_a\right)^{\beta^k/\theta^k}} \tag{78}$$

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A.4 The Government Possibility Frontier

Consider a voter who resides in area $a \in \mathcal{A}_j$ and chooses their preferred level of government spending per capita G_j. The system of equations implied by the housing market clearing and government balanced budget conditions is

$$\frac{\partial J_a}{\partial \log G_j} d\log G_j + \frac{\partial J_a}{\partial \log R_a} d\log R_a + \frac{\partial J_a}{\partial \log (1+\tau_j)} d\log (1+\tau_j) = 0 \tag{79}$$

$$\frac{\partial K_j}{\partial \log G_j} d\log G_j + \frac{\partial K_j}{\partial \log R_a} d\log R_a + \frac{\partial K_j}{\partial \log (1+\tau_j)} d\log (1+\tau_j) = 0 \tag{80}$$

where equation (80) must hold for every $j \in \mathcal{J}_a$. The goal of this section is to compute the partial derivatives required to solve this system in its general form. Recall that

$$J_a \equiv \lambda + (1+\eta)\log R_a + B_a - \log \sum_{k'} \pi^{k'} N_a^{k'} + \log (1+\tau_a) \tag{81}$$

$$K_j \equiv \log \tau_j + \log \sum_{a' \in \mathcal{A}_j} R_{a'} H_{a'} - \log G_j - \log \sum_{a' \in \mathcal{A}_j} N_{a'} \tag{82}$$

A.4.1 Sum of Exponentials

For any household type k, the partial derivatives of $\widetilde{\phi}^k$, i.e., the reciprocal of the sum of exponentials, are the following:

$$\frac{\partial \widetilde{\phi}^k}{\partial \log G_j} = -\left(\sum_{a'} \exp\left(v_{a'}^k/\theta^k\right)\right)^{-2} \left(\frac{\alpha_j^k}{\theta^k} \sum_{a' \in \mathcal{A}_j} \exp\left(v_{a'}^k/\theta^k\right)\right) \tag{83}$$

$$= -\frac{\alpha_j^k}{\theta^k} \widetilde{\phi}^k \frac{\sum_{a' \in \mathcal{A}_j} \exp\left(v_{a'}^k/\theta^k\right)}{\sum_{a'} \exp\left(v_{a'}^k/\theta^k\right)} = -\frac{\alpha_j^k}{\theta^k} \widetilde{\phi}^k S_j^k \tag{84}$$

$$\frac{\partial \widetilde{\phi}^k}{\partial \log R_a} = -\left(\sum_{a'} \exp\left(v_{a'}^k/\theta^k\right)\right)^{-2} \left(-\frac{\beta^k}{\theta^k} \exp\left(v_a^k/\theta^k\right)\right) \tag{85}$$

$$= \frac{\beta^k}{\theta^k} \widetilde{\phi}^k \frac{\exp\left(v_a^k/\theta^k\right)}{\sum_{a'} \exp\left(v_{a'}^k/\theta^k\right)} = \frac{\beta^k}{\theta^k} \widetilde{\phi}^k S_a^k \tag{86}$$

$$\frac{\partial \widetilde{\phi}^k}{\partial \log (1+\tau_j)} = -\left(\sum_{a'} \exp\left(v_{a'}^k/\theta^k\right)\right)^{-2} \left(-\frac{\beta^k}{\theta^k} (1+\tau_j) \sum_{a' \in \mathcal{A}_j} \frac{\exp\left(v_{a'}^k/\theta^k\right)}{1+\tau_{a'}}\right) \tag{87}$$

$$= \frac{\beta^k}{\theta^k} (1+\tau_j)\, \widetilde{\phi}^k \frac{\sum_{a' \in \mathcal{A}_j} \frac{\exp\left(v_{a'}^k/\theta^k\right)}{1+\tau_{a'}}}{\sum_{a'} \exp\left(v_{a'}^k/\theta^k\right)} \tag{88}$$

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A.4.2 Population by Area and Type

To begin with, for any household type k, area a, and jurisdiction $j \in \mathcal{J}_a$,

$$\frac{\partial N_a^k}{\partial \log G_j} = \frac{\partial N_a^k / \partial G_j}{\partial \log G_j / \partial G_j} = G_j \frac{\partial N_a^k}{\partial G_j} \tag{89}$$

$$= G_j \left(\frac{\partial \widetilde{\phi}^k}{\partial G_j} e^{\zeta^k} \frac{e^{\overline{a}_a^k/\theta^k} \Pi_{j' \in \mathcal{J}_a} G_{j'}^{\alpha_{j'}^k/\theta^k}}{R_a^{\beta^k/\theta^k} (1+\tau_a)^{\beta^k/\theta^k}} + \widetilde{\phi}^k e^{\zeta^k} \frac{e^{\overline{a}_a^k/\theta^k} \Pi_{j' \in \mathcal{J}_a} G_{j'}^{\alpha_{j'}^k/\theta^k}}{R_a^{\beta^k/\theta^k} (1+\tau_a)^{\beta^k/\theta^k}} \frac{\alpha_j^k}{\theta^k} G_j^{-1} \right) \tag{90}$$

$$= G_j \left(\frac{\partial \widetilde{\phi}^k}{\partial \log G_j} \frac{\partial \log G_j}{\partial G_j} e^{\zeta^k} \frac{e^{\overline{a}_a^k/\theta^k} \Pi_{j' \in \mathcal{J}_a} G_{j'}^{\alpha_j^k/\theta^k}}{R_a^{\beta^k/\theta^k} (1+\tau_a)^{\beta^k/\theta^k}} + N_a^k \frac{\alpha_j^k}{\theta^k} \frac{1}{G_j} \right) \tag{91}$$

$$= G_j \left(-\frac{\alpha_j^k}{\theta^k} \widetilde{\phi}^k S_j^k \frac{1}{G_j} e^{\zeta^k} \frac{e^{\overline{a}_a^k/\theta^k} \Pi_{j' \in \mathcal{J}_a} G_{j'}^{\alpha_j^k/\theta^k}}{R_a^{\beta^k/\theta^k} (1+\tau_a)^{\beta^k/\theta^k}} + N_a^k \frac{\alpha_j^k}{\theta^k} \frac{1}{G_j} \right) \tag{92}$$

$$= G_j \left(-\frac{\alpha_j^k}{\theta^k} S_j^k \frac{1}{G_j} N_a^k + N_a^k \frac{\alpha_j^k}{\theta^k} \frac{1}{G_j} \right) \tag{93}$$

$$= \left(-\frac{\alpha_j^k}{\theta^k} S_j^k N_a^k + N_a^k \frac{\alpha_j^k}{\theta^k} \right) \tag{94}$$

$$= \frac{\alpha_j^k}{\theta^k} N_a^k \left(1 - S_j^k\right) \tag{95}$$

Instead, for any household type k, area a, and jurisdiction $j \notin \mathcal{J}_a$,

$$\frac{\partial N_a^k}{\partial \log G_j} = -\frac{\alpha_j^k}{\theta^k} N_a^k S_j^k \tag{96}$$

In addition, for any household type k and area a,

$$\frac{\partial N_a^k}{\partial \log R_a} = \frac{\partial N_a^k / \partial R_a}{\partial \log R_a / \partial R_a} = R_a \frac{\partial N_a^k}{\partial R_a} \tag{97}$$

$$= R_a \left(\frac{\partial \widetilde{\phi}^k}{\partial R_a} e^{\zeta^k} \frac{e^{\overline{a}_a^k/\theta^k} \Pi_{j' \in \mathcal{J}_a} G_{j'}^{\alpha_{j'}^k/\theta^k}}{R_a^{\beta^k/\theta^k} (1+\tau_a)^{\beta^k/\theta^k}} - \widetilde{\phi}^k e^{\zeta^k} \frac{e^{\overline{a}_a^k/\theta^k} \Pi_{j' \in \mathcal{J}_a} G_{j'}^{\alpha_{j'}^k/\theta^k}}{R_a^{\beta^k/\theta^k} (1+\tau_a)^{\beta^k/\theta^k}} \frac{\beta^k}{\theta^k} R_a^{-1} \right) \tag{98}$$

$$= R_a \left(\frac{\partial \widetilde{\phi}^k}{\partial \log R_a} \frac{\partial \log R_a}{\partial R_a} e^{\zeta^k} \frac{e^{\overline{a}_a^k/\theta^k} \Pi_{j' \in \mathcal{J}_a} G_{j'}^{\alpha_j^k/\theta^k}}{R_a^{\beta^k/\theta^k} (1+\tau_a)^{\beta^k/\theta^k}} - N_a^k \frac{\beta^k}{\theta^k} \frac{1}{R_a} \right) \tag{99}$$

$$= R_a \left(\frac{\beta^k}{\theta^k} \widetilde{\phi}^k S_a^k \frac{1}{R_a} e^{\zeta^k} \frac{e^{\overline{a}_a^k/\theta^k} \Pi_{j' \in \mathcal{J}_a} G_{j'}^{\alpha_j^k/\theta^k}}{R_a^{\beta^k/\theta^k} (1+\tau_a)^{\beta^k/\theta^k}} - N_a^k \frac{\beta^k}{\theta^k} \frac{1}{R_a} \right) \tag{100}$$

$$= R_a \left(\frac{\beta^k}{\theta^k} S_a^k \frac{1}{R_a} N_a^k - N_a^k \frac{\beta^k}{\theta^k} \frac{1}{R_a} \right) \tag{101}$$

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$$= \left(\frac{\beta^k}{\theta^k} S_a^k N_a^k - N_a^k \frac{\beta^k}{\theta^k} \right) \tag{102}$$

$$= -\frac{\beta^k}{\theta^k} N_a^k \left(1 - S_a^k\right) \tag{103}$$

Instead, for any household type k and area $a' \neq a$,

$$\frac{\partial N_a^k}{\partial \log R_{a'}} = \frac{\beta^k}{\theta^k} N_{a'}^k S_{a'}^k \tag{104}$$

Finally, for any household type k, area a, and jurisdiction $j \in \mathcal{J}_a$,

$$\frac{\partial N_a^k}{\partial \log(1+\tau_j)} = \frac{\partial N_a^k / \partial (1+\tau_j)}{\partial \log(1+\tau_j) / \partial (1+\tau_j)} = (1+\tau_j) \frac{\partial N_a^k}{\partial (1+\tau_j)} \tag{105}$$

$$= \left(\frac{\partial \widetilde{\phi}^k}{\partial (1+\tau_j)} e^{\zeta^k} \frac{e^{\overline{a}_a^k/\theta^k} \Pi_{j' \in \mathcal{J}_a} G_{j'}^{\alpha_{j'}^k/\theta^k}}{R_a^{\beta^k/\theta^k} (1+\tau_a)^{\beta^k/\theta^k}} \right. \tag{106}$$

$$\left. - \widetilde{\phi}^k e^{\zeta^k} \frac{e^{\overline{a}_a^k/\theta^k} \Pi_{j' \in \mathcal{J}_a} G_{j'}^{\alpha_{j'}^k/\theta^k}}{R_a^{\beta^k/\theta^k} (1+\tau_a)^{\beta^k/\theta^k}} \frac{\beta^k}{\theta^k} (1+\tau_a)^{-1} \right) (1+\tau_j) \tag{107}$$

$$= \left(\frac{\partial \widetilde{\phi}^k}{\partial \log(1+\tau_j)} \frac{\partial \log(1+\tau_j)}{\partial (1+\tau_j)} e^{\zeta^k} \frac{e^{\overline{a}_a^k/\theta^k} \Pi_{j' \in \mathcal{J}_a} G_{j'}^{\alpha_j^k/\theta^k}}{R_a^{\beta^k/\theta^k} (1+\tau_a)^{\beta^k/\theta^k}} \right. \tag{108}$$

$$\left. - N_a^k \frac{\beta^k}{\theta^k} \frac{1}{1+\tau_a} \right) (1+\tau_j) \tag{109}$$

$$= \left(\frac{\beta^k}{\theta^k} (1+\tau_j) \widetilde{\phi}^k \frac{\sum_{a' \in \mathcal{A}_j} \frac{\exp\left(v_{a'}^k/\theta^k\right)}{1+\tau_{a'}}}{\sum_{a'} \exp\left(v_{a'}^k/\theta^k\right)} \frac{1}{1+\tau_j} e^{\zeta^k} \frac{e^{\overline{a}_a^k/\theta^k} \Pi_{j' \in \mathcal{J}_a} G_{j'}^{\alpha_j^k/\theta^k}}{R_a^{\beta^k/\theta^k} (1+\tau_a)^{\beta^k/\theta^k}} \right. \tag{110}$$

$$\left. - N_a^k \frac{\beta^k}{\theta^k} \frac{1}{1+\tau_a} \right) (1+\tau_j) \tag{111}$$

$$= (1+\tau_j) \left(\frac{\beta^k}{\theta^k} \frac{\sum_{a' \in \mathcal{A}_j} \frac{\exp\left(v_{a'}^k/\theta^k\right)}{1+\tau_{a'}}}{\sum_{a'} \exp\left(v_{a'}^k/\theta^k\right)} N_a^k - N_a^k \frac{\beta^k}{\theta^k} \frac{1}{1+\tau_a} \right) \tag{112}$$

$$= \frac{\beta^k}{\theta^k} N_a^k \left(\frac{\sum_{a' \in \mathcal{A}_j} \frac{1+\tau_j}{1+\tau_{a'}} \exp\left(v_{a'}^k/\theta^k\right)}{\sum_{a'} \exp\left(v_{a'}^k/\theta^k\right)} - \frac{1+\tau_j}{1+\tau_a} \right) \tag{113}$$

$$= -\frac{\beta^k}{\theta^k} N_a^k \left(\frac{1+\tau_j}{1+\tau_a} - \frac{\sum_{a' \in \mathcal{A}_j} \frac{1+\tau_j}{1+\tau_{a'}} \exp\left(v_{a'}^k/\theta^k\right)}{\sum_{a'} \exp\left(v_{a'}^k/\theta^k\right)} \right) \tag{114}$$

Instead, for any household type k, area a, and jurisdiction $j \notin \mathcal{J}_a$,

$$\frac{\partial N_a^k}{\partial \log(1+\tau_j)} = \frac{\beta^k}{\theta^k} N_a^k \frac{\sum_{a' \in \mathcal{A}_j} \frac{1+\tau_j}{1+\tau_{a'}} \exp\left(v_{a'}^k/\theta^k\right)}{\sum_{a'} \exp\left(v_{a'}^k/\theta^k\right)} \tag{115}$$

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A.4.3 Logged Population by Area and Type

To begin with, for any household type k, area a, and jurisdiction $j \in \mathcal{J}_a$,

$$\frac{\partial \log N_a^k}{\partial \log G_j} = \frac{\partial \log N_a^k}{\partial N_a^k} \frac{\partial N_a^k}{\partial \log G_j} = \frac{1}{N_a^k} \frac{\alpha_j^k}{\theta^k} N_a^k \left(1 - S_j^k\right) = \frac{\alpha_j^k}{\theta^k} \left(1 - S_j^k\right) \tag{116}$$

Instead, for any household type k, area a, and jurisdiction $j \notin \mathcal{J}_a$,

$$\frac{\partial \log N_a^k}{\partial \log G_j} = -\frac{\alpha_j^k}{\theta^k} S_j^k \tag{117}$$

In addition, for any household type k and area a,

$$\frac{\partial \log N_a^k}{\partial \log R_a} = \frac{\partial \log N_a^k}{\partial N_a^k} \frac{\partial N_a^k}{\partial \log R_a} = -\frac{1}{N_a^k} \frac{\beta^k}{\theta^k} N_a^k \left(1 - S_a^k\right) = -\frac{\beta^k}{\theta^k} \left(1 - S_a^k\right) \tag{118}$$

Instead, for any household type k and area $a' \neq a$,

$$\frac{\partial \log N_a^k}{\partial \log R_{a'}} = \frac{\beta^k}{\theta^k} S_{a'}^k \tag{119}$$

Finally, for any $j \in \mathcal{J}_a$,

$$\frac{\partial \log N_a^k}{\partial \log (1+\tau_j)} = \frac{\partial \log N_a^k}{\partial N_a^k} \frac{\partial N_a^k}{\partial \log (1+\tau_j)} \tag{120}$$

$$= -\frac{1}{N_a^k} \frac{\beta^k}{\theta^k} N_a^k \left(\frac{1+\tau_j}{1+\tau_a} - \frac{\sum_{a' \in \mathcal{A}_j} \frac{1+\tau_j}{1+\tau_{a'}} \exp\left(v_{a'}^k/\theta^k\right)}{\sum_{a'} \exp\left(v_{a'}^k/\theta^k\right)} \right) \tag{121}$$

$$= -\frac{\beta^k}{\theta^k} \left(\frac{1+\tau_j}{1+\tau_a} - \frac{\sum_{a' \in \mathcal{A}_j} \frac{1+\tau_j}{1+\tau_{a'}} \exp\left(v_{a'}^k/\theta^k\right)}{\sum_{a'} \exp\left(v_{a'}^k/\theta^k\right)} \right) \quad \text{for any } j \in \mathcal{J}_a \tag{122}$$

Instead, for any household type k, area a, and jurisdiction $j \notin \mathcal{J}_a$,

$$\frac{\partial \log N_a^k}{\partial \log (1+\tau_j)} = \frac{\beta^k}{\theta^k} \frac{\sum_{a' \in \mathcal{A}_j} \frac{1+\tau_j}{1+\tau_{a'}} \exp\left(v_{a'}^k/\theta^k\right)}{\sum_{a'} \exp\left(v_{a'}^k/\theta^k\right)} \tag{123}$$

A.4.4 Logged Population by Area

To begin with, for any area a and jurisdiction $j \in \mathcal{J}_a$,

$$\frac{\partial \log N_a}{\partial \log G_j} = \frac{\partial \log N_a}{\partial N_a} \frac{\partial N_a}{\partial \log G_j} = \frac{1}{N_a} \frac{\partial \sum_{k'} N_a^{k'}}{\partial \log G_j} \tag{124}$$

$$= \frac{1}{N_a} \sum_{k'} \frac{\partial N_a^{k'}}{\partial \log G_j} = \frac{1}{N_a} \sum_{k'} \frac{\alpha_j^{k'}}{\theta^k} N_a^{k'} \left(1 - S_j^{k'}\right) \tag{125}$$

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Instead, for any area a and jurisdiction $j \notin \mathcal{J}_a$,

$$\frac{\partial \log N_a}{\partial \log G_j} = -\frac{1}{N_a} \sum_{k'} \frac{\alpha_j^{k'}}{\theta^k} N_a^{k'} S_j^{k'} \tag{126}$$

In addition, for any area a,

$$\frac{\partial \log N_a}{\partial \log R_a} = \frac{\partial \log N_a}{\partial N_a} \frac{\partial N_a}{\partial \log R_a} = \frac{1}{N_a} \frac{\partial \sum_{k'} N_a^{k'}}{\partial \log R_a} \tag{127}$$

$$= \frac{1}{N_a} \sum_{k'} \frac{\partial N_a^{k'}}{\partial \log R_a} = -\frac{1}{N_a} \sum_{k'} \frac{\beta^{k'}}{\theta^{k'}} N_a^{k'} \left(1 - S_a^{k'}\right) \tag{128}$$

Instead, for any area $a' \neq a$,

$$\frac{\partial \log N_a}{\partial \log R_{a'}} = \frac{1}{N_a} \sum_{k'} \frac{\beta^{k'}}{\theta^{k'}} N_{a'}^{k'} S_{a'}^{k'} \tag{129}$$

Finally, for any area a and jurisdiction $j \in \mathcal{J}_a$,

$$\frac{\partial \log N_a}{\partial \log (1+\tau_j)} = \frac{\partial \log N_a}{\partial N_a} \frac{\partial N_a}{\partial \log (1+\tau_j)} = \frac{1}{N_a} \frac{\partial \sum_{k'} N_a^{k'}}{\partial \log (1+\tau_j)} \tag{130}$$

$$= \frac{1}{N_a} \sum_{k'} \frac{\partial N_a^{k'}}{\partial \log (1+\tau_j)} \tag{131}$$

$$= -\frac{1}{N_a} \sum_{k'} \frac{\beta^{k'}}{\theta^{k'}} N_{a'}^{k'} \left(\frac{1+\tau_j}{1+\tau_a} - \frac{\sum_{a' \in \mathcal{A}_j} \frac{1+\tau_j}{1+\tau_{a'}} \exp\left(v_{a'}^{k'}/\theta^{k'}\right)}{\sum_{a'} \exp\left(v_{a'}^{k'}/\theta^{k'}\right)} \right) \tag{132}$$

Instead, for any area a and jurisdiction $j \notin \mathcal{J}_a$,

$$\frac{\partial \log N_a}{\partial \log (1+\tau_j)} = \frac{1}{N_a} \sum_{k'} \frac{\beta^{k'}}{\theta^{k'}} N_{a'}^{k'} \frac{\sum_{a' \in \mathcal{A}_j} \frac{1+\tau_j}{1+\tau_{a'}} \exp\left(v_{a'}^{k'}/\theta^{k'}\right)}{\sum_{a'} \exp\left(v_{a'}^{k'}/\theta^{k'}\right)} \tag{133}$$

A.4.5 System of Equations for the Government Possibility Frontier

For any area a, the partial derivatives associated with the market clearing condition J_a are

$$\frac{\partial J_a}{\partial \log G_j} = -\left(\sum_{k'} \pi^{k'} N_a^{k'}\right)^{-1} \sum_{k'} \pi^{k'} \frac{\alpha_j^{k'}}{\theta^{k'}} N_a^{k'} \left(1 - S_j^{k'}\right) \quad \text{for any } j \in \mathcal{J}_a \tag{134}$$

$$\frac{\partial J_a}{\partial \log G_j} = \left(\sum_{k'} \pi^{k'} N_a^{k'}\right)^{-1} \sum_{k'} \pi^{k'} \frac{\alpha_j^{k'}}{\theta^{k'}} N_a^{k'} S_j^{k'} \quad \text{for any } j \notin \mathcal{J}_a \tag{135}$$

$$\frac{\partial J_a}{\partial \log R_a} = 1 + \eta + \left(\sum_{k'} \pi^{k'} N_a^{k'}\right)^{-1} \sum_{k'} \pi^{k'} \frac{\beta^{k'}}{\theta^{k'}} N_a^{k'} \left(1 - S_a^{k'}\right) \tag{136}$$

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$$\frac{\partial J_a}{\partial \log R_{a'}} = -\left(\sum_{k'} \pi^{k'} N_a^{k'}\right)^{-1} \sum_{k'} \pi^{k'} \frac{\beta^{k'}}{\theta^{k'}} N_a^{k'} S_a^{k'} \quad \text{for any } a' \neq a \tag{137}$$

$$\frac{\partial J_a}{\partial \log(1+\tau_j)} = \frac{1+\tau_j}{1+\tau_a} + \left(\sum_{k'} \pi^{k'} N_a^{k'}\right)^{-1} \tag{138}$$

$$\sum_{k'} \pi^{k'} \frac{\beta^{k'}}{\theta^{k'}} N_a^{k'} \left(\frac{1+\tau_j}{1+\tau_a} - \frac{\sum_{a' \in \mathcal{A}_j} \frac{1+\tau_j}{1+\tau_{a'}} \exp\left(v_{a'}^{k'}/\theta^{k'}\right)}{\sum_{a'} \exp\left(v_{a'}^{k'}/\theta^{k}\right)}\right) \quad \text{for any } j \in \mathcal{J}_a \tag{139}$$

$$\frac{\partial J_a}{\partial \log(1+\tau_j)} = -\left(\sum_{k'} \pi^{k'} N_a^{k'}\right)^{-1} \tag{140}$$

$$\sum_{k'} \pi^{k'} \frac{\beta^{k'}}{\theta^{k'}} N_a^{k'} \frac{\sum_{a' \in \mathcal{A}_j} \frac{1+\tau_j}{1+\tau_{a'}} \exp\left(v_{a'}^{k'}/\theta^{k'}\right)}{\sum_{a'} \exp\left(v_{a'}^{k'}/\theta^{k'}\right)} \quad \text{for any } j \notin \mathcal{J}_a \tag{141}$$

For any jurisdiction j, the partial derivatives associated with the balanced budget condition K_j are

$$\frac{\partial K_j}{\partial \log G_j} = -1 - \frac{1}{\sum_{a' \in \mathcal{A}_j} N_{a'}} \sum_{a' \in \mathcal{A}_j} \sum_{k'} \frac{\alpha_j^{k'}}{\theta^{k'}} N_{a'}^{k'} \left(1 - S_j^{k'}\right) \tag{142}$$

$$\frac{\partial K_j}{\partial \log G_{j'}} = \frac{1}{\sum_{a' \in \mathcal{A}_j} N_{a'}} \sum_{a' \in \mathcal{A}_j} \sum_{k'} \frac{\alpha_{j'}^{k'}}{\theta^{k'}} N_{a'}^{k'} S_{j'}^{k'} \quad \text{for any } j' \neq j \tag{143}$$

$$\frac{\partial K_j}{\partial \log R_a} = \frac{(1+\eta) R_a H_a}{\sum_{a' \in \mathcal{A}_j} R_{a'} H_{a'}} + \frac{1}{\sum_{a' \in \mathcal{A}_j} N_{a'}} \sum_{k'} \frac{\beta^{k'}}{\theta^{k'}} N_a^{k'} \left(1 - S_a^{k'}\right) \quad \text{for any } a \in \mathcal{A}_j \tag{144}$$

$$\frac{\partial K_j}{\partial \log R_a} = -\frac{1}{\sum_{a' \in \mathcal{A}_j} N_{a'}} \sum_{k'} \frac{\beta^{k'}}{\theta^{k'}} N_a^{k'} S_a^{k'} \quad \text{for any } a \notin \mathcal{A}_j \tag{145}$$

$$\frac{\partial K_j}{\partial \log(1+\tau_j)} = \frac{1+\tau_j}{\tau_j} \tag{146}$$

$$+ \frac{1}{\sum_{a' \in \mathcal{A}_j} N_{a'}} \sum_{a' \in \mathcal{A}_j} \sum_{k'} \frac{\beta^{k'}}{\theta^{k'}} N_{a'}^{k'} \left(\frac{1+\tau_j}{1+\tau_a} - \frac{\sum_{a' \in \mathcal{A}_j} \frac{1+\tau_j}{1+\tau_{a'}} \exp\left(v_{a'}^{k'}/\theta^{k'}\right)}{\sum_{a'} \exp\left(v_{a'}^{k'}/\theta^{k'}\right)}\right) \tag{147}$$

$$\frac{\partial K_j}{\partial \log(1+\tau_{j'})} = -\frac{1}{\sum_{a' \in \mathcal{A}_j} N_{a'}} \sum_{a' \in \mathcal{A}_j} \sum_{k'} \frac{\beta^{k'}}{\theta^{k'}} N_{a'}^{k'} \frac{\sum_{a' \in \mathcal{A}_j} \frac{1+\tau_{j'}}{1+\tau_{a'}} \exp\left(v_{a'}^{k'}/\theta^{k'}\right)}{\sum_{a'} \exp\left(v_{a'}^{k'}/\theta^{k'}\right)} \quad \text{for any } j' \neq j \tag{148}$$

A.4.6 Partial Derivatives with Myopic Voting

The assumption of myopic voting entails that voters perceive jurisdiction boundaries as fixed and do not account for the mobility implications of a change in local expenditures and taxes. As a consequence, all of the terms involving a partial derivative of N_a^k are set to zero. The

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resulting partial derivatives from the previous section change as follows. For any area a,

$$\frac{\partial J_a}{\partial \log G_j} = 0 \quad \text{for any } j \in \mathcal{J}_a \tag{149}$$

$$\frac{\partial J_a}{\partial \log G_j} = 0 \quad \text{for any } j \notin \mathcal{J}_a \tag{150}$$

$$\frac{\partial J_a}{\partial \log R_a} = 1 + \eta \tag{151}$$

$$\frac{\partial J_a}{\partial \log R_{a'}} = 0 \quad \text{for any } a' \neq a \tag{152}$$

$$\frac{\partial J_a}{\partial \log (1+\tau_j)} = \frac{1+\tau_j}{1+\tau_a} \quad \text{for any } j \in \mathcal{J}_a \tag{153}$$

$$\frac{\partial J_a}{\partial \log (1+\tau_j)} = 0 \quad \text{for any } j \notin \mathcal{J}_a \tag{154}$$

In addition, for any jurisdiction j,

$$\frac{\partial K_j}{\partial \log G_j} = -1 \tag{155}$$

$$\frac{\partial K_j}{\partial \log G_{j'}} = 0 \quad \text{for any } j' \neq j \tag{156}$$

$$\frac{\partial K_j}{\partial \log R_a} = \frac{(1+\eta)\, R_a H_a}{\sum_{a' \in \mathcal{A}_j} R_{a'} H_{a'}} \equiv (1+\eta)\, \Psi_{aj} \quad \text{for any } a \in \mathcal{A}_j \tag{157}$$

$$\frac{\partial K_j}{\partial \log R_a} = 0 \quad \text{for any } a \notin \mathcal{A}_j \tag{158}$$

$$\frac{\partial K_j}{\partial \log (1+\tau_j)} = \frac{1+\tau_j}{\tau_j} \tag{159}$$

$$\frac{\partial K_j}{\partial \log (1+\tau_{j'})} = 0 \quad \text{for any } j' \neq j \tag{160}$$

A.4.7 The Slope of the Government Possibility Frontier

Consider a voter who resides in area a and chooses their preferred level of government spending in jurisdiction $j \in \mathcal{J}_a$. Let $\mathcal{J}_a = \left\{1, \ldots, j, \ldots, \bar{j}\right\}$. In matrix form, the system of equations implied by the budget balance and housing market clearing conditions is

$$\begin{bmatrix} J_{ar} & J_{a\tau_1} & \ldots & J_{a\tau_j} & \ldots & J_{a\tau_{\bar{j}}} \\ K_{1r} & K_{1\tau_1} & \ldots & K_{1\tau_j} & \ldots & K_{1\tau_{\bar{j}}} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ K_{jr} & K_{j\tau_1} & \ldots & K_{j\tau_j} & \ldots & K_{j\tau_{\bar{j}}} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ K_{\bar{j}r} & K_{\bar{j}\tau_1} & \ldots & K_{\bar{j}\tau_j} & \ldots & K_{\bar{j}\tau_{\bar{j}}} \end{bmatrix} \begin{bmatrix} dr_a/dg_j \\ d\tau_1/dg_j \\ \vdots \\ d\tau_j/dg_j \\ \vdots \\ d\tau_{\bar{j}}/dg_j \end{bmatrix} = \begin{bmatrix} -J_{ag_j} \\ -K_{1g_j} \\ \vdots \\ -K_{jg_j} \\ \vdots \\ -K_{\bar{j}g_j} \end{bmatrix} \tag{161}$$

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where the matrix of known coefficients is the Jacobian associated with the housing market clearing and balanced budget conditions. In addition, the unknowns are defined as $dg_j \equiv d\log G_j$, $dr_a \equiv d\log R_a$, and $d\tau_j \equiv d\log(1+\tau_j)$.

A.4.8 The Slope of the GPF with Myopic Voting

Under the assumption of myopic voting, the system of equations in (161) becomes

$$\begin{bmatrix} J_{ar} & J_{a\tau_1} & \dots & J_{a\tau_j} & \dots & J_{a\tau_{\bar{j}}} \\ K_{1r} & K_{1\tau_1} & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ K_{jr} & 0 & \dots & K_{j\tau_j} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ K_{\bar{j}r} & 0 & \dots & 0 & \dots & K_{\bar{j}\tau_{\bar{j}}} \end{bmatrix} \begin{bmatrix} dr_a/dg_j \\ d\tau_1/dg_j \\ \vdots \\ d\tau_j/dg_j \\ \vdots \\ d\tau_{\bar{j}}/dg_j \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ -K_{jg_j} \\ \vdots \\ 0 \end{bmatrix} \tag{162}$$

To derive a closed-form expression for the solution to this system, consider the balanced budget equation for any jurisdiction $j' \in \mathcal{J}_a$:

$$K_{j'r}\frac{dr_a}{dg_j} + K_{j'\tau_{j'}}\frac{d\tau_{j'}}{dg_j} = -K_{j'g_j} \iff \frac{d\tau_{j'}}{dg_j} = -\frac{K_{j'g_j}}{K_{j'\tau_{j'}}} - \frac{K_{j'r}}{K_{j'\tau_{j'}}}\frac{dr_a}{dg_j} \tag{163}$$

Plugging this expression into the housing market clearing condition yields

$$J_{ar}\frac{dr_a}{dg_j} + \sum_{j'\in\mathcal{J}_a} J_{a\tau_{j'}}\frac{d\tau_{j'}}{dg_j} = 0 \iff J_{ar}\frac{dr_a}{dg_j} + \sum_{j'\in\mathcal{J}_a} J_{a\tau_{j'}}\left(-\frac{K_{j'g_j}}{K_{j'\tau_{j'}}} - \frac{K_{j'r}}{K_{j'\tau_{j'}}}\frac{dr_a}{dg_j}\right) = 0 \tag{164}$$

$$\iff J_{ar}\frac{dr_a}{dg_j} - \sum_{j'\in\mathcal{J}_a}\frac{J_{a\tau_{j'}}K_{j'g_j}}{K_{j'\tau_{j'}}} - \sum_{j'\in\mathcal{J}_a}\frac{J_{a\tau_{j'}}K_{j'r}}{K_{j'\tau_{j'}}}\frac{dr_a}{dg_j} = 0 \tag{165}$$

$$\iff \left(J_{ar} - \sum_{j'\in\mathcal{J}_a}\frac{J_{a\tau_{j'}}K_{j'r}}{K_{j'\tau_{j'}}}\right)\frac{dr_a}{dg_j} = \sum_{j'\in\mathcal{J}_a}\frac{J_{a\tau_{j'}}K_{j'g_j}}{K_{j'\tau_{j'}}} \tag{166}$$

$$\iff \frac{dr_a}{dg_j} = \left(J_{ar} - \sum_{j'\in\mathcal{J}_a}\frac{J_{a\tau_{j'}}K_{j'r}}{K_{j'\tau_{j'}}}\right)^{-1}\sum_{j'\in\mathcal{J}_a}\frac{J_{a\tau_{j'}}K_{j'g_j}}{K_{j'\tau_{j'}}} \tag{167}$$

Finally, the slope of the property tax rate levied by jurisdiction j' is

$$\frac{d\tau_{j'}}{dg_j} = -\frac{K_{j'g_j}}{K_{j'\tau_{j'}}} - \frac{K_{j'r}}{K_{j'\tau_{j'}}}\frac{dr_a}{dg_j} \tag{168}$$

$$= -\frac{K_{j'g_j}}{K_{j'\tau_{j'}}} - \frac{K_{j'r}}{K_{j'\tau_{j'}}}\left(J_{ar} - \sum_{\ell\in\mathcal{J}_a}\frac{J_{a\tau_\ell}K_{\ell r}}{K_{\ell\tau_\ell}}\right)^{-1}\sum_{\ell\in\mathcal{J}_a}\frac{J_{a\tau_\ell}K_{\ell g_j}}{K_{\ell\tau_\ell}} \tag{169}$$

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The previously computed partial derivatives can now be used to determine the total derivative of the rental rate of housing with respect to government spending:

$$\frac{d\log R_a}{d\log G_j} = \left(J_{ar} - \sum_{j'\in\mathcal{J}_a} \frac{J_{a\tau_{j'}}K_{j'r}}{K_{j'\tau_{j'}}}\right)^{-1} \sum_{j'\in\mathcal{J}_a} \frac{J_{a\tau_{j'}}K_{j'g_j}}{K_{j'\tau_{j'}}} \tag{170}$$

$$= \left(1+\eta - \sum_{j'\in\mathcal{J}_a} \frac{\frac{1+\tau_{j'}}{1+\tau_a}(1+\eta)\Psi_{aj'}}{\frac{1+\tau_{j'}}{\tau_{j'}}}\right)^{-1} \sum_{j'\in\mathcal{J}_a} \frac{\frac{1+\tau_{j'}}{1+\tau_a}(-1)\mathbb{I}[j'=j]}{\frac{1+\tau_{j'}}{\tau_{j'}}} \tag{171}$$

$$= -\left(1+\eta - \sum_{j'\in\mathcal{J}_a} \frac{\tau_{j'}}{1+\tau_a}(1+\eta)\Psi_{aj'}\right)^{-1} \frac{\tau_j}{1+\tau_a} \tag{172}$$

$$= -\frac{1}{1+\eta}\left(1 - \frac{\sum_{j'\in\mathcal{J}_a}\Psi_{aj'}\tau_{j'}}{1+\tau_a}\right)^{-1} \frac{\tau_j}{1+\tau_a} \tag{173}$$

$$= -\frac{1}{1+\eta}\left(\frac{1+\tau_a - \sum_{j'\in\mathcal{J}_a}\Psi_{aj'}\tau_{j'}}{1+\tau_a}\right)^{-1} \frac{\tau_j}{1+\tau_a} \tag{174}$$

$$= -\frac{1}{1+\eta}\left(\frac{\tau_j}{1+\tau_a - \sum_{j'\in\mathcal{J}_a}\Psi_{aj'}\tau_{j'}}\right) \tag{175}$$

Similarly, the total derivative of jurisdiction $j' \neq j$'s property tax rate with respect to government spending per capita is

$$\frac{d\log(1+\tau_{j'})}{d\log G_j} = -\frac{K_{j'g_j}}{K_{j'\tau_{j'}}} - \frac{K_{j'r}}{K_{j'\tau_{j'}}}\frac{d\log R_a}{d\log G_j} \tag{176}$$

$$= -\frac{(1+\eta)\Psi_{aj'}}{\frac{1+\tau_{j'}}{\tau_{j'}}}\left(-\frac{1}{1+\eta}\frac{\tau_j}{1+\tau_a - \sum_{\ell\in\mathcal{J}_a}\Psi_{a\ell}\tau_\ell}\right) \tag{177}$$

$$= \frac{\Psi_{aj'}\tau_{j'}}{1+\tau_{j'}}\left(\frac{\tau_j}{1+\tau_a - \sum_{\ell\in\mathcal{J}_a}\Psi_{a\ell}\tau_\ell}\right) \tag{178}$$

Instead, for jurisdiction j,

$$\frac{d\log(1+\tau_j)}{d\log G_j} = -\frac{K_{jg_j}}{K_{j\tau_j}} - \frac{K_{jr}}{K_{j\tau_j}}\frac{d\log R_a}{d\log G_j} \tag{179}$$

$$= \frac{1}{\frac{1+\tau_j}{\tau_j}} - \frac{(1+\eta)\Psi_{aj}}{\frac{1+\tau_j}{\tau_j}}\left(-\frac{1}{1+\eta}\frac{\tau_j}{1+\tau_a - \sum_{\ell\in\mathcal{J}_a}\Psi_{a\ell}\tau_\ell}\right) \tag{180}$$

$$= \frac{\tau_j}{1+\tau_j} + \frac{\Psi_{aj}\tau_j}{1+\tau_j}\left(\frac{\tau_j}{1+\tau_a - \sum_{\ell\in\mathcal{J}_a}\Psi_{a\ell}\tau_\ell}\right) \tag{181}$$

A.5 Preferred Property Tax Rates

The goal of this section is to derive the property tax rate preferred by any household type k residing in any area a for any jurisdiction j.

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A.5.1 First-Order Conditions

Consider a voter in area a choosing their preferred level of government spending per capita on the public good provided by jurisdiction $j \in \mathcal{J}_a$. The derivative of household i's indirect utility function with respect to government spending is

$$\frac{dV_{ia}}{d \log G_j} = \alpha_j^k - \beta^k \frac{d \log R_a}{d \log G_j} - \beta^k \sum_{j' \in \mathcal{J}_a} \frac{1+\tau_{j'}}{1+\tau_a} \frac{d \log (1+\tau_{j'})}{d \log G_j} \tag{182}$$

As in equation (13), the first-order condition associated with the implied maximization problem is

$$\alpha_j^k = \beta^k \frac{d \log R_a}{d \log G_j}\bigg|_{G_j = G_{ja}^k} + \beta^k \sum_{j' \in \mathcal{J}_a} \frac{1+\tau_{j'}}{1+\tau_a} \frac{d \log (1+\tau_{j'})}{d \log G_j}\bigg|_{G_j = G_{ja}^k} \tag{183}$$

Let us maintain the assumption that voters are myopic. First, the property tax component of the marginal cost of increasing government spending is

$$\sum_{j' \in \mathcal{J}_a} \frac{1+\tau_{j'}}{1+\tau_a} \frac{d \log (1+\tau_{j'})}{d \log G_j} = \sum_{j' \in \mathcal{J}_a} \frac{1+\tau_{j'}}{1+\tau_a} \frac{\Psi_{aj'}\tau_{j'}}{1+\tau_{j'}} \left(\frac{\tau_j}{1+\tau_a - \sum_{j' \in \mathcal{J}_a} \Psi_{aj'}\tau_{j'}} \right) \tag{184}$$

$$+ \frac{1+\tau_j}{1+\tau_a} \frac{\tau_j}{1+\tau_j} \tag{185}$$

$$= \frac{\tau_j}{1+\tau_a} + \sum_{j' \in \mathcal{J}_a} \frac{\Psi_{aj'}\tau_{j'}}{1+\tau_a} \left(\frac{\tau_j}{1+\tau_a - \sum_{j' \in \mathcal{J}_a} \Psi_{aj'}\tau_{j'}} \right) \tag{186}$$

$$= \frac{\tau_j}{1+\tau_a} + \frac{\sum_{j' \in \mathcal{J}_a} \Psi_{aj'}\tau_{j'}}{1+\tau_a} \left(\frac{\tau_j}{1+\tau_a - \sum_{j' \in \mathcal{J}_a} \Psi_{aj'}\tau_{j'}} \right) \tag{187}$$

Replacing the two derivatives with the expressions derived in the previous section yields

$$\alpha_j^k = -\beta^k \frac{1}{1+\eta} \left(\frac{\tau_j}{1+\tau_a - \sum_{j' \in \mathcal{J}_a} \Psi_{aj'}\tau_{j'}} \right) \tag{188}$$

$$+ \beta^k \frac{\sum_{j' \in \mathcal{J}_a} \Psi_{aj'}\tau_{j'}}{1+\tau_a} \left(\frac{\tau_j}{1+\tau_a - \sum_{j' \in \mathcal{J}_a} \Psi_{aj'}\tau_{j'}} \right) + \beta^k \frac{\tau_j}{1+\tau_a} \tag{189}$$

$$= \beta^k \left(\frac{\tau_j}{1+\tau_a - \sum_{j' \in \mathcal{A}_j} \Psi_{aj'}\tau_{j'}} \right) \left(\frac{\sum_{j' \in \mathcal{J}_a} \Psi_{aj'}\tau_{j'}}{1+\tau_a} - \frac{1}{1+\eta} \right) + \beta^k \frac{\tau_j}{1+\tau_a} \tag{190}$$

This first-order condition is evaluated at $\tau_j = \tau_{ja}^k$, jurisdiction j's property tax rate preferred by type-k households residing in area $a \in \mathcal{A}_j$.

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A.5.2 Preferred Property Tax Rates

The set of preferred property tax rates for type-k households in area a is the solution to the system of $|\mathcal{J}_a|$ equations implied by the first-order conditions in (190). The preferred property tax rate for jurisdiction $j \in \mathcal{J}_a$ is therefore

$$\tau_{ja}^k = \frac{\alpha_j^k (1+\eta)}{\beta^k \eta - (1+\eta) \sum_{j' \in \mathcal{J}_a} \alpha_{j'}^k (1 - \Psi_{aj'})} \tag{191}$$

The numerator of τ_{ja}^k is positive because, by assumption, all of the elements of $\left\{\alpha_j^k\right\}_{j,k}$ are positive and the elasticity of housing supply η is positive. However, without further restrictions, the denominator may be negative, possibly yielding illogically valued tax rates. In the worst-case scenario, $\Psi_{aj'} \to 0$ for all $j' \in \mathcal{J}_a$, which would imply that area a does not belong to any of the jurisdictions in $\mathcal{J}_a$. In this case,

$$\lim_{\Psi_{aj'} \to 0 \ \forall j' \in \mathcal{J}_a} \left(\beta^k \eta - (1+\eta) \sum_{j' \in \mathcal{J}_a} \alpha_{j'}^k (1 - \Psi_{aj'}) \right) = \beta^k \eta - (1+\eta) \sum_{j' \in \mathcal{J}_a} \alpha_{j'}^k \tag{192}$$

which is positive provided that

$$\beta^k \eta - (1+\eta) \sum_{j' \in \mathcal{J}_a} \alpha_{j'}^k > 0 \iff \frac{\sum_{j' \in \mathcal{J}_a} \alpha_{j'}^k}{\beta^k} < \frac{\eta}{1+\eta} \tag{193}$$

To conclude, if the inequality in (193) is true, the optimal property tax rate τ_{ja}^k is positive for any set of housing expenditure shares $\{\Psi_{aj'}\}_{j' \in \mathcal{J}_a}$.

A.5.3 Second-Order Conditions

The goal of this section is to determine whether τ_{ja}^k is indeed a maximizer of V_{ia}. Replacing equation (190) into equation (194) yields a compact expression for the first derivative of the indirect utility:

$$\frac{dV_{ia}}{d \log G_j} = \alpha_j^k - \beta^k \left(\frac{\tau_j}{1 + \tau_a - \sum_{j' \in \mathcal{J}_a} \Psi_{aj'} \tau_{j'}} \right) \left(\frac{\sum_{j' \in \mathcal{J}_a} \Psi_{aj'} \tau_{j'}}{1+\tau_a} - \frac{1}{1+\eta} \right) - \beta^k \frac{\tau_j}{1+\tau_a} \tag{194}$$

By two applications of the chain rule, the second derivative of the indirect utility is

$$\frac{d^2 V_{ia}}{d (\log G_j)^2} = \sum_{j' \in \mathcal{J}_a} \frac{d \frac{dV_{ia}}{d \log G_j}}{d\tau_{j'}} \frac{d\tau_{j'}}{d \log G_j} \tag{195}$$

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$$= \sum_{j' \in \mathcal{J}_a} \frac{d \frac{dV_{ia}}{d \log G_j}}{d\tau_{j'}} \frac{d\tau_{j'}}{d \log(1+\tau_{j'})} \frac{d \log(1+\tau_{j'})}{d \log G_j} \tag{196}$$

$$= \sum_{j' \in \mathcal{J}_a} \frac{d \frac{dV_{ia}}{d \log G_j}}{d\tau_{j'}} (1+\tau_{j'}) \frac{d \log(1+\tau_{j'})}{d \log G_j} \tag{197}$$

As shown in (178), the derivative of the property tax rate in a different jurisdiction is

$$(1+\tau_{j'}) \frac{d \log(1+\tau_{j'})}{d \log G_j} = \tau_j \left(\frac{\Psi_{aj'} \tau_{j'}}{1 + \sum_{\ell \in \mathcal{J}_a} (1-\Psi_{a\ell}) \tau_\ell} \right) > 0 \tag{198}$$

Instead, as shown in (181), the derivative of the property tax rate in the same jurisdiction is

$$(1+\tau_j) \frac{d \log(1+\tau_j)}{d \log G_j} = \tau_j + \Psi_{aj} \tau_j \left(\frac{\tau_j}{1 + \sum_{\ell \in \mathcal{J}_a} (1-\Psi_{a\ell}) \tau_\ell} \right) \tag{199}$$

$$= \tau_j \left(1 + \frac{\Psi_{aj} \tau_j}{1 + \sum_{\ell \in \mathcal{J}_a} (1-\Psi_{a\ell}) \tau_\ell} \right) \tag{200}$$

$$= \tau_j \left(\frac{1 + \sum_{\ell \in \mathcal{J}_a} (1-\Psi_{a\ell}) \tau_\ell + \Psi_{aj} \tau_j}{1 + \sum_{\ell \in \mathcal{J}_a} (1-\Psi_{a\ell}) \tau_\ell} \right) > 0 \tag{201}$$

Moreover, the derivative of $\frac{dV_{ia}}{d \log G_j}$ with respect to the tax rate in a different jurisdiction is

$$\frac{d \frac{dV_{ia}}{d \log G_j}}{d\tau_{j'}} = \left(\sum_{\ell \in \mathcal{J}_a} \sum_{m \in \mathcal{J}_a} (1-\Psi_{a\ell})(1-\Psi_{am}) \tau_\ell \tau_m + 2 \sum_{\ell \in \mathcal{J}_a} (1-\Psi_{a\ell}) \tau_\ell + 1 \right)^{-1} \tag{202}$$

$$\beta^k \eta (1-\Psi_{aj'}) \tau_j \tag{203}$$

$$= \left(1 + \sum_{\ell \in \mathcal{J}_a} (1-\Psi_{a\ell}) \tau_\ell \right)^{-2} \beta^k \eta (1-\Psi_{aj'}) \tau_j > 0 \tag{204}$$

Finally, the derivative of $\frac{dV_{ia}}{d \log G_j}$ with respect to the tax rate in the same jurisdiction is

$$\frac{d \frac{dV_{ia}}{d \log G_j}}{d\tau_j} = - \left(\sum_{\ell \in \mathcal{J}_a} \sum_{m \in \mathcal{J}_a} (1-\Psi_{a\ell})(1-\Psi_{am}) \tau_\ell \tau_m + 2 \sum_{\ell \in \mathcal{J}_a} (1-\Psi_{a\ell}) \tau_\ell + 1 \right)^{-1} \tag{205}$$

$$\beta^k \eta \left(1 + \sum_{\ell \in \mathcal{J}_a \setminus \{j\}} (1-\Psi_{a\ell}) \tau_\ell \right) \tag{206}$$

$$= - \left(1 + \sum_{\ell \in \mathcal{J}_a} (1-\Psi_{a\ell}) \tau_\ell \right)^{-2} \beta^k \eta \left(1 + \sum_{\ell \in \mathcal{J}_a \setminus \{j\}} (1-\Psi_{a\ell}) \tau_\ell \right) < 0 \tag{207}$$

To keep notation compact, I define the following terms:

$$\triangle_{j'} \equiv \frac{d \frac{dV_{ia}}{d \log G_j}}{d\tau_{j'}} (1+\tau_{j'}) \frac{d \log(1+\tau_{j'})}{d \log G_j} \qquad \square_j \equiv \frac{d \frac{dV_{ia}}{d \log G_j}}{d\tau_j} (1+\tau_j) \frac{d \log(1+\tau_j)}{d \log G_j} \tag{208}$$

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Combining previous expressions, the $j' \neq j$ term in the summation on line (197) is

$$\triangle_{j'} = \left(1 + \sum_{\ell \in \mathcal{J}_a} (1 - \Psi_{a\ell}) \tau_\ell\right)^{-2} \beta^k \eta (1 - \Psi_{aj'}) \tau_j \tau_j \left(\frac{\Psi_{aj'} \tau_{j'}}{1 + \sum_{\ell \in \mathcal{J}_a} (1 - \Psi_{a\ell}) \tau_\ell}\right) \tag{209}$$

$$= \frac{\beta^k \eta (1 - \Psi_{aj'}) \tau_j \tau_j \Psi_{aj'} \tau_{j'}}{\left(1 + \sum_{\ell \in \mathcal{J}_a} (1 - \Psi_{a\ell}) \tau_\ell\right)^3} \tag{210}$$

$$= \frac{\beta^k \eta (1 - \Psi_{aj'}) \Psi_{aj'} \tau_{j'} \tau_j^2}{\left(1 + \sum_{\ell \in \mathcal{J}_a} (1 - \Psi_{a\ell}) \tau_\ell\right)^3} > 0 \tag{211}$$

Similarly, the $j' = j$ term in the summation on line (197) is

$$\square_j = -\left(1 + \sum_{\ell \in \mathcal{J}_a} (1 - \Psi_{a\ell}) \tau_\ell\right)^{-2} \beta^k \eta \left(1 + \sum_{\ell \in \mathcal{J}_a \setminus \{j\}} (1 - \Psi_{a\ell}) \tau_\ell\right) \tag{212}$$

$$\tau_j \left(\frac{1 + \sum_{\ell \in \mathcal{J}_a} (1 - \Psi_{a\ell}) \tau_\ell + \Psi_{aj} \tau_j}{1 + \sum_{\ell \in \mathcal{J}_a} (1 - \Psi_{a\ell}) \tau_\ell}\right) \tag{213}$$

$$= -\left(1 + \sum_{\ell \in \mathcal{J}_a} (1 - \Psi_{a\ell}) \tau_\ell\right)^{-3} \beta^k \eta \left(1 + \sum_{\ell \in \mathcal{J}_a \setminus \{j\}} (1 - \Psi_{a\ell}) \tau_\ell\right) \tag{214}$$

$$\tau_j \left(1 + \sum_{\ell \in \mathcal{J}_a} (1 - \Psi_{a\ell}) \tau_\ell + \Psi_{aj} \tau_j\right) \tag{215}$$

$$= -\frac{\beta^k \eta \left(1 + \sum_{\ell \in \mathcal{J}_a \setminus \{j\}} (1 - \Psi_{a\ell}) \tau_\ell\right) \left(1 + \sum_{\ell \in \mathcal{J}_a} (1 - \Psi_{a\ell}) \tau_\ell + \Psi_{aj} \tau_j\right) \tau_j}{\left(1 + \sum_{\ell \in \mathcal{J}_a} (1 - \Psi_{a\ell}) \tau_\ell\right)^3} \tag{216}$$

$$= -\frac{\beta^k \eta \left(1 + \sum_{\ell \in \mathcal{J}_a \setminus \{j\}} (1 - \Psi_{a\ell}) \tau_\ell\right) \left(1 + \sum_{\ell \in \mathcal{J}_a \setminus \{j\}} (1 - \Psi_{a\ell}) \tau_\ell + \tau_j\right) \tau_j}{\left(1 + \sum_{\ell \in \mathcal{J}_a} (1 - \Psi_{a\ell}) \tau_\ell\right)^3} \tag{217}$$

$$= -\frac{\beta^k \eta \left(1 + \sum_{\ell \in \mathcal{J}_a \setminus \{j\}} (1 - \Psi_{a\ell}) \tau_\ell\right)^2 \tau_j}{\left(1 + \sum_{\ell \in \mathcal{J}_a} (1 - \Psi_{a\ell}) \tau_\ell\right)^3} \tag{218}$$

$$- \frac{\beta^k \eta \left(1 + \sum_{\ell \in \mathcal{J}_a \setminus \{j\}} (1 - \Psi_{a\ell}) \tau_\ell\right) \tau_j^2}{\left(1 + \sum_{\ell \in \mathcal{J}_a} (1 - \Psi_{a\ell}) \tau_\ell\right)^3} < 0 \tag{219}$$

Thus, the summation on line (197) reduces to

$$\sum_{j' \in \mathcal{J}_a \setminus \{j\}} \triangle_{j'} + \square_j = \frac{\sum_{\ell \in \mathcal{J}_a \setminus \{j\}} \beta^k \eta (1 - \Psi_{a\ell}) \Psi_{a\ell} \tau_\ell \tau_j^2}{\left(1 + \sum_{\ell \in \mathcal{J}_a} (1 - \Psi_{a\ell}) \tau_\ell\right)^3} \tag{220}$$

$$- \frac{\beta^k \eta \left(1 + \sum_{\ell \in \mathcal{J}_a \setminus \{j\}} (1 - \Psi_{a\ell}) \tau_\ell\right) \tau_j^2}{\left(1 + \sum_{\ell \in \mathcal{J}_a} (1 - \Psi_{a\ell}) \tau_\ell\right)^3} \tag{221}$$

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$$-\frac{\beta^k\eta\left(1+\sum_{\ell\in\mathcal{J}_a\setminus\{j\}}\left(1-\Psi_{a\ell}\right)\tau_\ell\right)^2\tau_j}{\left(1+\sum_{\ell\in\mathcal{J}_a}\left(1-\Psi_{a\ell}\right)\tau_\ell\right)^3} \tag{222}$$

Focusing on the terms on lines (220) and (221):

$$\frac{\sum_{\ell\in\mathcal{J}_a\setminus\{j\}}\beta^k\eta\left(1-\Psi_{a\ell}\right)\Psi_{a\ell}\tau_\ell\tau_j^2}{\left(1+\sum_{\ell\in\mathcal{J}_a}\left(1-\Psi_{a\ell}\right)\tau_\ell\right)^3}-\frac{\beta^k\eta\left(1+\sum_{\ell\in\mathcal{J}_a\setminus\{j\}}\left(1-\Psi_{a\ell}\right)\tau_\ell\right)\tau_j^2}{\left(1+\sum_{\ell\in\mathcal{J}_a}\left(1-\Psi_{a\ell}\right)\tau_\ell\right)^3} \tag{223}$$

$$=\frac{\beta^k\eta\tau_j^2}{\left(1+\sum_{\ell\in\mathcal{J}_a}\left(1-\Psi_{a\ell}\right)\tau_\ell\right)^3}\left(\sum_{\ell\in\mathcal{J}_a\setminus\{j\}}\left(1-\Psi_{a\ell}\right)\Psi_{a\ell}\tau_\ell-1-\sum_{\ell\in\mathcal{J}_a\setminus\{j\}}\left(1-\Psi_{a\ell}\right)\tau_\ell\right) \tag{224}$$

$$=\frac{\beta^k\eta\tau_j^2}{\left(1+\sum_{\ell\in\mathcal{J}_a}\left(1-\Psi_{a\ell}\right)\tau_\ell\right)^3}\left(-1+\sum_{\ell\in\mathcal{J}_a\setminus\{j\}}\left(1-\Psi_{a\ell}\right)\left(\Psi_{a\ell}-1\right)\tau_\ell\right) \tag{225}$$

$$=\frac{\beta^k\eta\tau_j^2}{\left(1+\sum_{\ell\in\mathcal{J}_a}\left(1-\Psi_{a\ell}\right)\tau_\ell\right)^3}\left(-1-\sum_{\ell\in\mathcal{J}_a\setminus\{j\}}\left(1-\Psi_{a\ell}\right)^2\tau_\ell\right)<0 \tag{226}$$

Because the term on line (222) is negative, $\sum_{j'\in\mathcal{J}_a\setminus\{j\}}\triangle_{j'}+\square_j$ is negative too, implying that the indirect utility V_{ia} is a strictly concave function of $\log G_j$. Thus, τ_{ja}^k attains the unique global maximum of V_{ia}.

A.5.4 Comparative Statics

In this section, I check how the preferred tax rate varies as a function of parameter values. I focus on the preference for government spending per capita α_j^k, the preference for housing space β^k, and the elasticity of housing supply η. As shown in equation (191), the property tax rate preferred by type-k households residing in area a for jurisdiction j is

$$\tau_{ja}^k=\frac{\alpha_j^k\left(1+\eta\right)}{\beta^k\eta-\left(1+\eta\right)\sum_{j'\in\mathcal{J}_a}\alpha_{j'}^k\left(1-\Psi_{aj'}\right)} \tag{227}$$

First, consider the derivative of τ_{ja}^k with respect to α_j^k:

$$\frac{d\tau_{ja}^k}{d\alpha_j^k}=\frac{\left(1+\eta\right)\left(\beta^k\eta-\left(1+\eta\right)\sum_{j'\in\mathcal{J}_a}\alpha_{j'}^k\left(1-\Psi_{aj'}\right)\right)+\alpha_j^k\left(1+\eta\right)^2\left(1-\Psi_{aj}\right)}{\left(\beta^k\eta-\left(1+\eta\right)\sum_{j'\in\mathcal{J}_a}\alpha_{j'}^k\left(1-\Psi_{aj'}\right)\right)^2} \tag{228}$$

which is positive provided that the inequality in (193) is true. Second, the derivative of τ_{ja}^k with respect to β^k is

$$\frac{d\tau_{ja}^k}{d\beta^k}=-\frac{\alpha_j^k\left(1+\eta\right)\eta}{\left(\beta^k\eta-\left(1+\eta\right)\sum_{j'\in\mathcal{J}_a}\alpha_{j'}^k\left(1-\Psi_{aj'}\right)\right)^2} \tag{229}$$

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which is negative. Third, consider the derivative of τ_{ja}^k with respect to η:

$$\frac{d\tau_{ja}^k}{d\eta} = \frac{\alpha_j^k \left(\beta^k \eta - (1+\eta) \sum_{j' \in \mathcal{J}_a} \alpha_{j'}^k \left(1 - \Psi_{aj'}\right)\right) - \alpha_j^k \left(1+\eta\right) \left(\beta^k - \sum_{j' \in \mathcal{J}_a} \alpha_{j'}^k \left(1 - \Psi_{aj'}\right)\right)}{\left(\beta^k \eta - (1+\eta) \sum_{j' \in \mathcal{J}_a} \alpha_{j'}^k \left(1 - \Psi_{aj'}\right)\right)^2} \tag{230}$$

$$= \frac{\alpha_j^k \beta^k \eta - \alpha_j^k \left(1+\eta\right) \beta^k}{\left(\beta^k \eta - (1+\eta) \sum_{j' \in \mathcal{J}_a} \alpha_{j'}^k \left(1 - \Psi_{aj'}\right)\right)^2} \tag{231}$$

$$= -\frac{\alpha_j^k \beta^k}{\left(\beta^k \eta - (1+\eta) \sum_{j' \in \mathcal{J}_a} \alpha_{j'}^k \left(1 - \Psi_{aj'}\right)\right)^2} \tag{232}$$

which is negative too. Finally, the derivative of τ_{ja}^k with respect to the preference for government spending in a different jurisdiction $\alpha_{j'}^k$, with $j' \neq j$, is

$$\frac{d\tau_{ja}^k}{d\alpha_{j'}^k} = -\frac{\alpha_j^k \left(1+\eta\right)^2 \left(1 - \Psi_{aj'}\right)}{\left(\beta^k \eta - (1+\eta) \sum_{\ell \in \mathcal{J}_a} \alpha_\ell^k \left(1 - \Psi_{a\ell}\right)\right)^2} \tag{233}$$

which is again negative.

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